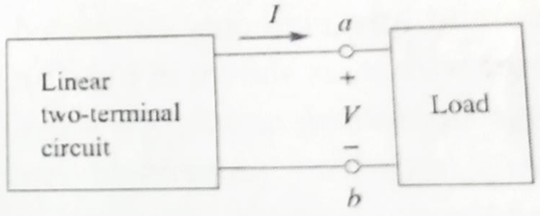
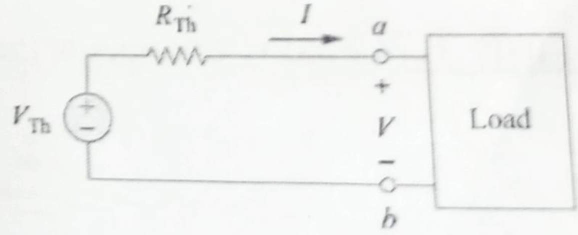
➢ Thevenin's Theorem

Thevenin's theorem states that **a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}**, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

According to Thevenin's theorem, the linear circuit in Fig. (a) can be replaced by that in Fig. (b). The circuit to the left of the terminals in Fig.(b) is known as the ***Thevenin equivalent circuit***; it was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.



(a)



(b)

The main challenge is how to find the Thevenin equivalent voltage and resistance. Thevenin's theorem is very important in circuit analysis. It helps simplify a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

- To apply this idea in finding the Thevenin resistance, we need to consider two cases.

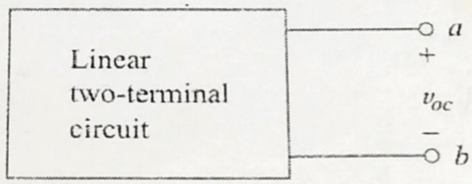
■ **CASE 1**: *If the network has no dependent source:*

(1) Find the Thevenin source voltage by removing the load resistor from the original circuit and calculating voltage across the open connection points where the load resistor used to be.

(2) Find the Thevenin resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.

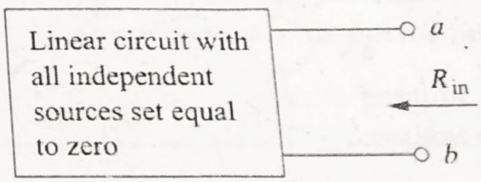
(3) Draw the Thevenin equivalent circuit, with the Thevenin voltage source in series with the Thevenin resistance. The load resistor re-attaches between the two open points of the equivalent circuit.

(4) Analyze voltage and current for the load resistor following the rules for series circuits.



$V_{Th} = v_{oc}$

(a)



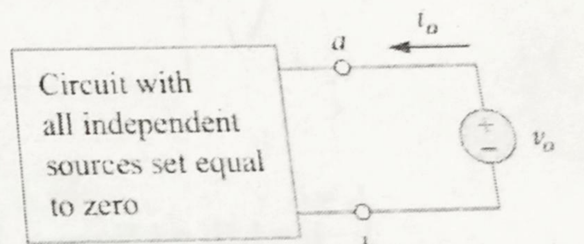
$R_{Th} = R_{in}$

(b)

If the network has no dependent sources, we turn off all independent sources. is the input resistance of the network looking between terminals a and b, as shown in Fig.(b)

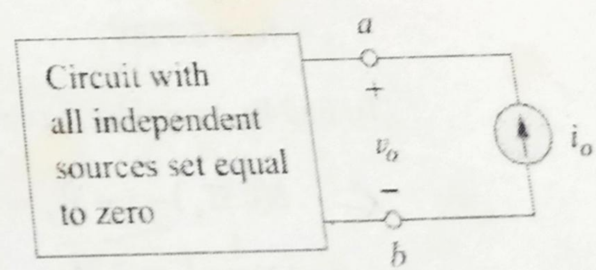
■ **CASE 2**: *If the network has dependent sources:*

If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_o at terminals a and b and determine the resulting current i_o. Then, $R_{Th} = v_o/i_o$, as shown in Fig.(a). Alternatively, we may insert a current source i_o at terminals a-b as shown in Fig.(b) and find the terminal voltage v_o. Again $R_{Th} = v_o/i_o$. Either of the two approaches will give the same result. In either approach, we may assume any value of v_o and i_o. For example, we may use $v_o = 1$ V or $i_o = 1$ A, or even use unspecified values of v_o or i_o.



$$R_{Th} = \frac{v_o}{i_o}$$

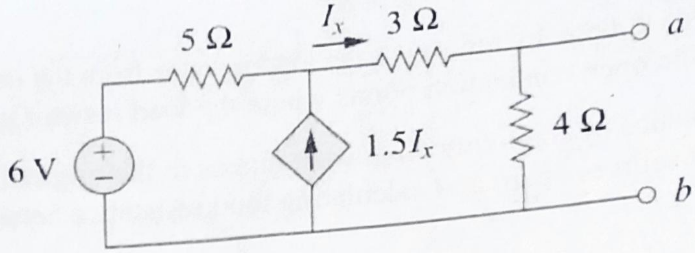
(a)



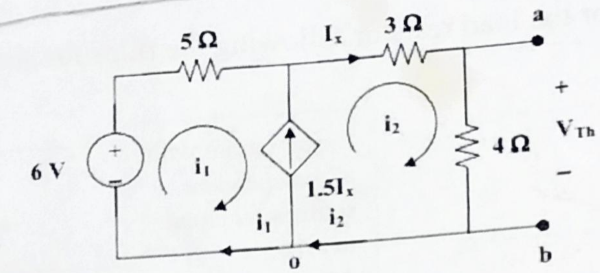
$$R_{Th} = \frac{v_o}{i_o}$$

(b)

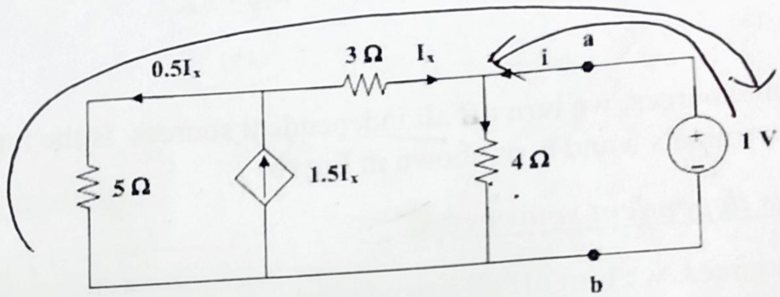
Example 1.8: Find the Thevenin equivalent circuit of the circuit in Fig. to the left of the terminals.



Solution: To find V_{Th}, consider the circuit in Fig.



(a)



(b)

$I_x = i_2$

$i_2 - i_1 = 1.5I_x = 1.5i_2 \longrightarrow i_2 = -2i_1$ (1)

For the supermesh, $-6 + 5i_1 + 7i_2 = 0$ (2) $4+3=7$

From (1) and (2), $i_2 = 4/(3)$A $= 1.33$A

$V_{Th} = 4i_2 = 5.333$V

To find R_{Th}, consider the circuit in Fig. (b). Applying KVL around the outer loop,

$5(0.5I_x) - 1 - 3I_x = 0 \longrightarrow I_x = -2$

$i = \frac{1}{4} - I_x = 2.25$

$I_x - \frac{1}{4} + i = 0$

$\Rightarrow i = \frac{1}{4} - I_x$

$R_{Th} = \frac{1}{i} = \frac{1}{2.25} = 444.4\ \text{m}\Omega$

➢ Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in Fig.(a) can be replaced by the one in Fig.(b)

- Steps to follow for Norton's Theorem:

(1) Find the Norton source current by removing the load resistor from the original circuit and calculating current through a short (wire) jumping across the open connection points where the load resistor used to be.

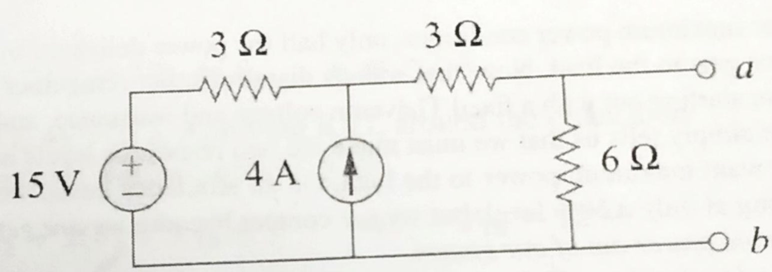
(2) Find the Norton resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.

(3) Draw the Norton equivalent circuit, with the Norton current source in parallel with the Norton resistance. The load resistor re-attaches between the two open points of the equivalent circuit.

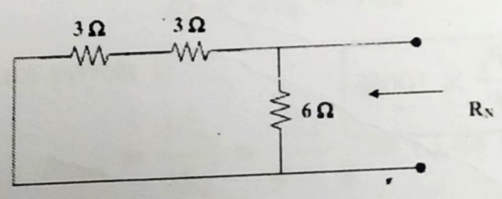
(4) Analyze voltage and current for the load resistor following the rules for parallel circuits.

❖ The Thevenin and Norton equivalent circuits are related by a source transformation.

Example 1.9: Find the Norton equivalent circuit for the circuit in Fig. 4.42, at terminals a-b.

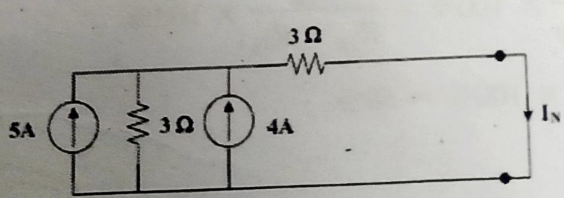


Solution:



(a)

$I = V/R = 15/3 = 5$



(b)

From Fig. (a), $R_N = (3+3)\|6 = 3\ \Omega$

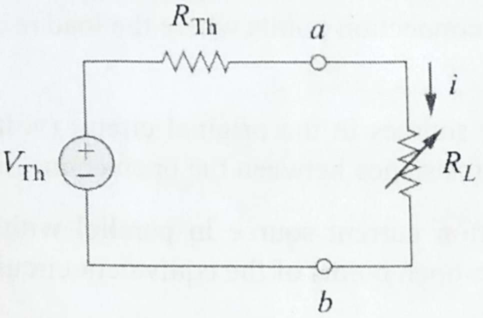
From Fig. (b), $I_N = \frac{1}{2}(5+4) = 4.5\text{A}$

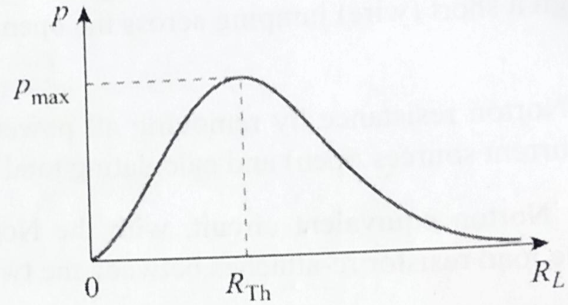
➢ Maximum Power Transfer

The Maximum Power Transfer Theorem states that the maximum amount of power will be dissipated by a load resistance if it is equal to the Thevenin or Norton resistance of the network supplying power.

"Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$)."

$$p = i^2 R_L = \left(\frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L}\right)^2 R_L$$





For the given circuit above, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched in Fig.We notice from Fig. that the power is small for small or large values of R_L but maximum for some value of R_L between 0 and infinity. We now want to show that this maximum power occurs when R_L is equal to R_{Th}. This is known as the R_{Th} maximum power theorem.

Under maximum power conditions, only half the power delivered by the source gets to the load. Now, that sounds disastrous, but remember that we are starting out with a fixed Thévenin voltage and resistance, and the above simply tells us that we must make the two resistance levels equal if we want maximum power to the load. On an efficiency basis, we are working at only a 50% level. but we are content because *we are getting maximum power out of our system.*

The dc operating efficiency is defined as the ratio of the power delivered to the load (P_L) to the power delivered by the source (P_s). That is,

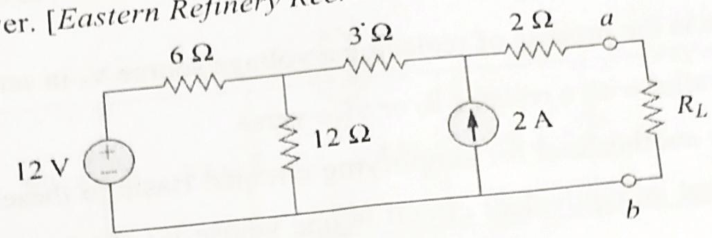
$$\boxed{\eta\% = \frac{P_L}{P_s} \times 100\%}$$

For the situation where $R_L = R_{Th}$,

$$\eta\% = \frac{I_L^2 R_L}{I_L^2 R_T} \times 100\% = \frac{R_L}{R_T} \times 100\% = \frac{R_{Th}}{R_{Th} + R_{Th}} \times 100\%$$

$$= \frac{R_{Th}}{2R_{Th}} \times 100\% = \frac{1}{2} \times 100\% = \mathbf{50\%}$$

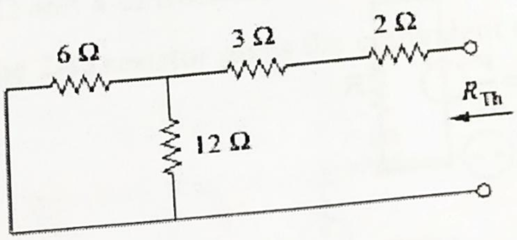
Example 1.10: Find the value of R_L for maximum power transfer in the circuit of Fig. below. Find the maximum power. [*Eastern Refinery Recruitment Exam*]



Solution:

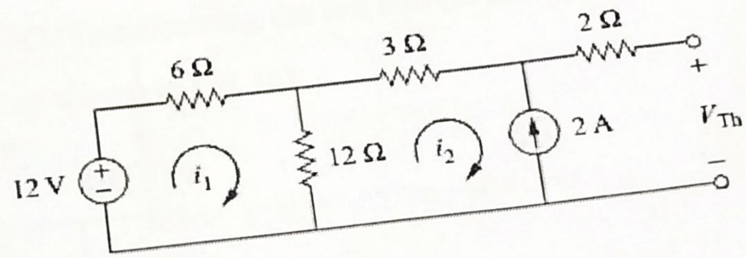
We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals a-b. To get R_{Th}, we use the circuit in Fig. and obtain

$$R_{Th} = 2 + 3 + 6 \,\|\, 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$



(a)

Finding R_{Th}



(b)

Finding V_{Th}

To get V_{Th}, we consider the circuit in Fig. (b). Applying mesh analysis gives

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\ \text{A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22\ \text{V}$$

For maximum power transfer,

$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\ \text{W}$$

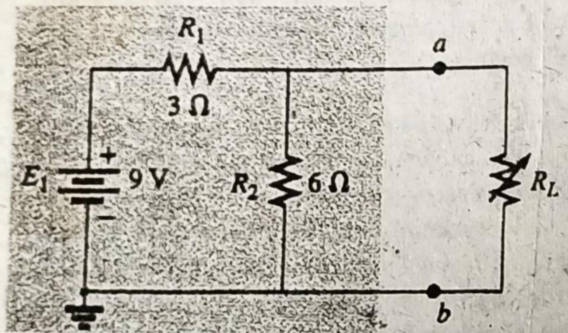


FIG. 9.26
Example 9.6.

EXAMPLE 9.6 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.26. Then find the current through R_L for values of 2 Ω, 10 Ω, and 100 Ω.

Solution:

Steps 1 and 2: These produce the network in Fig. 9.27. Note that the load resistor R_L has been removed and the two "holding" terminals have been defined as *a* and *b*.

Steps 3: Replacing the voltage source E_1 with a short-circuit equivalent yields the network in Fig. 9.28(a), where

$$R_{Th} = R_1 \| R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = 2\ \Omega$$

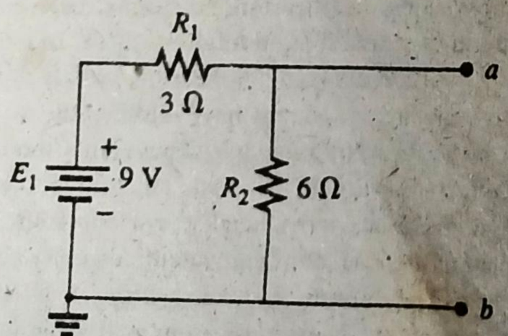


FIG. 9.27
Identifying the terminals of particular importance when applying Thévenin's theorem.

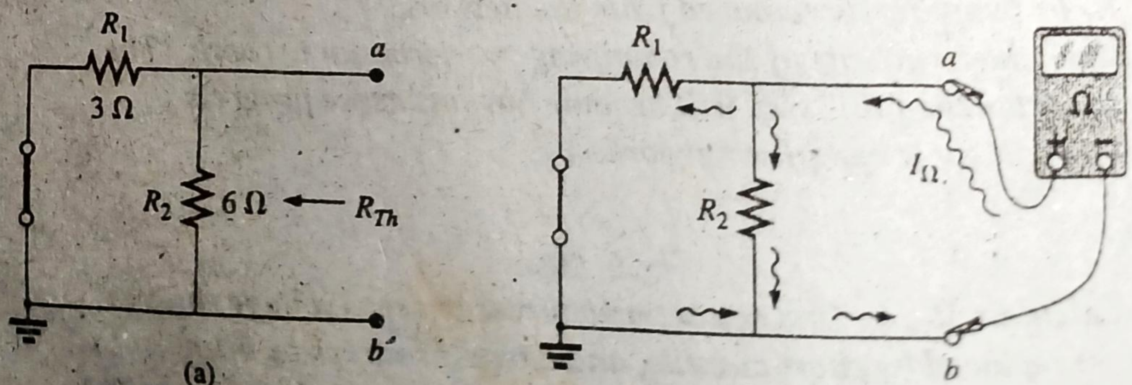


FIG. 9.28
Determining R_{Th} for the network in Fig. 9.27.

Th

The importance of the two marked terminals now begins to surface. They are the two terminals across which the Thévenin resistance is measured. It is no longer the total resistance as seen by the source, as determined in the majority of problems of Chapter 7. If some difficulty develops when determining R_{Th} with regard to whether the resistive elements are in series or parallel, consider recalling that the ohmmeter sends out a trickle current into a resistive combination and senses the level of the resulting voltage to establish the measured resistance level. In Fig. 9.28(b), the trickle current of the ohmmeter approaches the network through terminal a, and when it reaches the junction of R_1 and R_2, it splits as shown. The fact that the trickle current splits and then recombines at the lower node reveals that the resistors are in parallel as far as the ohmmeter reading is concerned. In essence, the path of the sensing current of the ohmmeter has revealed how the resistors are connected to the two terminals of interest and how the Thévenin resistance should be determined. Remember this as you work through the various examples in this section.

Step 4: Replace the voltage source (Fig. 9.29). For this case, the open-circuit voltage E_{Th} is the same as the voltage drop across the 6 Ω resistor. Applying the voltage divider rule,

$$E_{Th} = \frac{R_2E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = 6\ \text{V}$$

It is particularly important to recognize that E_{Th} is the open-circuit potential between points a and b. Remember that an open circuit can have any voltage across it, but the current must be zero. In fact, the current through any element in series with the open circuit must be zero also. The use of a voltmeter to measure E_{Th} appears in Fig. 9.30. Note that it is placed directly across the resistor R_2 since E_{Th} and V_{R_2} are in parallel.

Step 5 (Fig. 9.31):

$$\boxed{I_L = \frac{E_{Th}}{R_{Th} + R_L}}$$

$$R_L = 2\ \Omega: \quad I_L = \frac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$$R_L = 10\ \Omega: \quad I_L = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$$R_L = 100\ \Omega: \quad I_L = \frac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.06\ A}$$

If Thévenin's theorem were unavailable, each change in R_L would require that the entire network in Fig. 9.26 be reexamined to find the new value of R_L.

EXAMPLE 9.7 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.32.

Solution:

Steps 1 and 2: See Fig. 9.33.

Step 3: See Fig. 9.34. The current source has been replaced with an open-circuit equivalent, and the resistance determined between terminals a and b.

In this case, an ohmmeter connected between terminals a and b sends out a sensing current that flows directly through R_1 and R_2 (at the

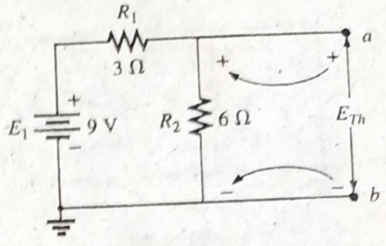


FIG. 9.29
Determining E_{Th} for the network in Fig. 9.27.

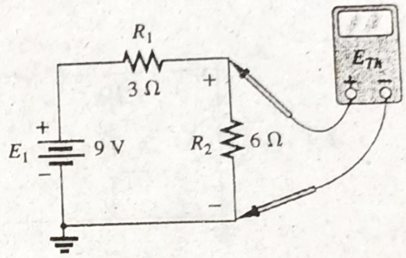


FIG. 9.30
Measuring E_{Th} for the network in Fig. 9.27.

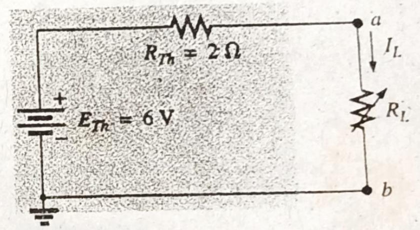


FIG. 9.31
Substituting the Thévenin equivalent circuit for the network external to R_L in Fig. 9.26.

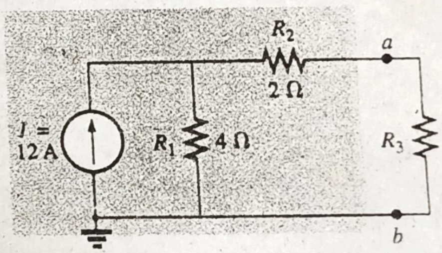


FIG. 9.32
Example 9.7.

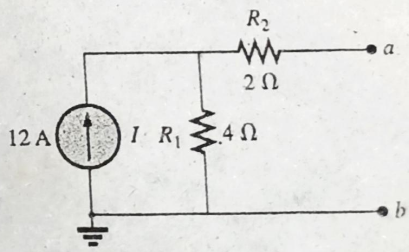


FIG. 9.33
Establishing the terminals of particular interest for the network in Fig. 9.32.

same level). The result is that R_1 and R_2 are in series and the Thévenin resistance is the sum of the two.

$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = \mathbf{6\ \Omega}$$

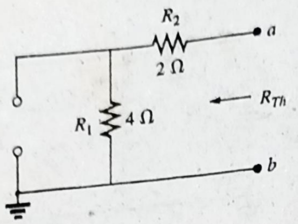


FIG. 9.34
Determining R_{Th} for the network in Fig. 9.33.

Step 4: See Fig. 9.35. In this case, since an open circuit exists between the two marked terminals, the current is zero between these terminals and through the 2 Ω resistor. The voltage drop across R_2 is, therefore,

$$V_2 = I_2R_2 = (0)R_2 = 0\ \text{V}$$

and

$$E_{Th} = V_1 = I_1R_1 = IR_1 = (12\ \text{A})(4\ \Omega) = \mathbf{48\ V}$$

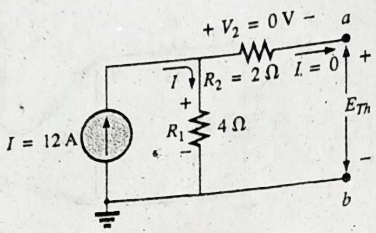


FIG. 9.35
Determining E_{Th} for the network in Fig. 9.33.

Step 5: See Fig. 9.36.

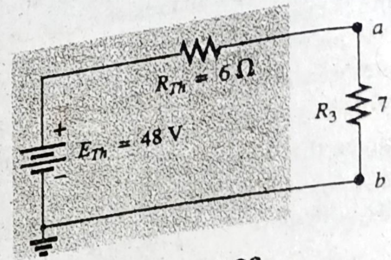


FIG. 9.36
Substituting the Thévenin equivalent circuit in the network external to the resistor R_3 in Fig. 9.32.

EXAMPLE 9.8 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.37. Note in this example that there is no need for the section of the network to be preserved to be at the "end" of the configuration.

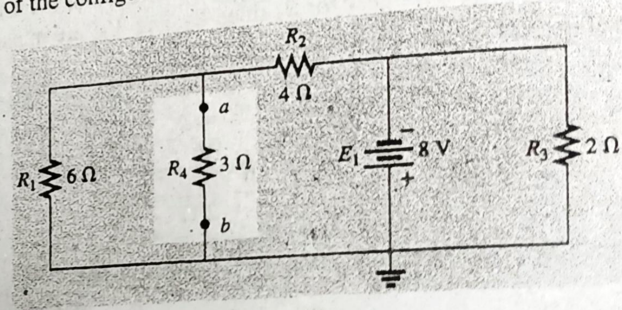


FIG. 9.37
Example 9.8.

Solution:

Steps 1 and 2: See Fig. 9.38.

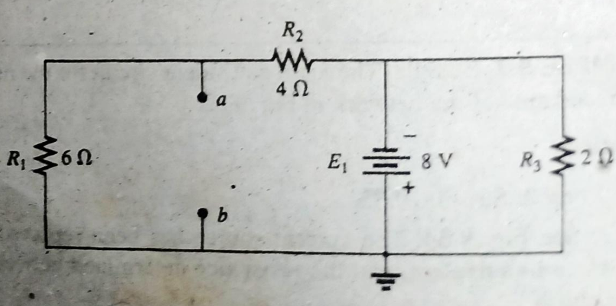


FIG. 9.38
Identifying the terminals of particular interest for the network in Fig. 9.37.

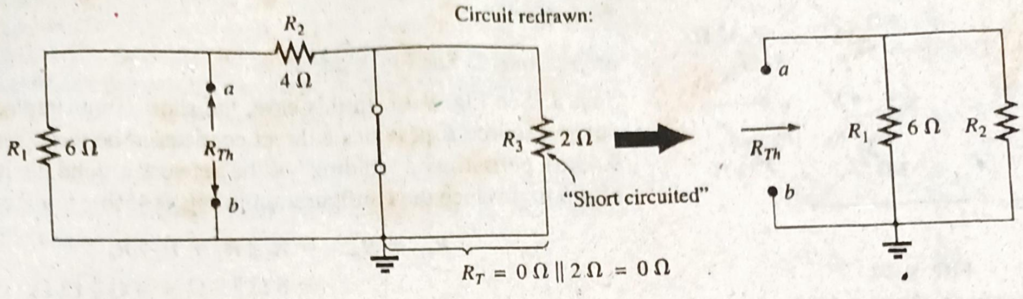


FIG. 9.39
Determining R_{Th} for the network in Fig. 9.38.

ep 3: See Fig. 9.39. Steps 1 and 2 are relatively easy to apply, but ow we must be careful to "hold" onto the terminals a and b as the hévenin resistance and voltage are determined. In Fig. 9.39, all the emaining elements turn out to be in parallel, and the network can be edrawn as shown.

$$R_{Th} = R_1 \parallel R_2 = \frac{(6\,\Omega)(4\,\Omega)}{6\,\Omega + 4\,\Omega} = \frac{24\,\Omega}{10} = \mathbf{2.4\,\Omega}$$

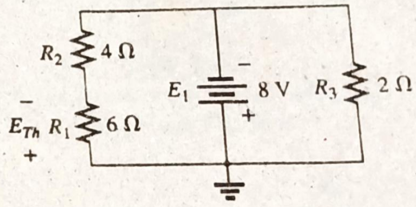


FIG. 9.41
Network of Fig. 9.40 redrawn.

ep 4: See Fig. 9.40. In this case, the network can be redrawn as shown Fig. 9.41. Since the voltage is the same across parallel elements, the ltage across the series resistors R_1 and R_2 is E_1, or 8 V. Applying the ltage divider rule,

$$E_{Th} = \frac{R_1 E_1}{R_1 + R_2} = \frac{(6\,\Omega)(8\,\text{V})}{6\,\Omega + 4\,\Omega} = \frac{48\,\text{V}}{10} = \mathbf{4.8\,V}$$

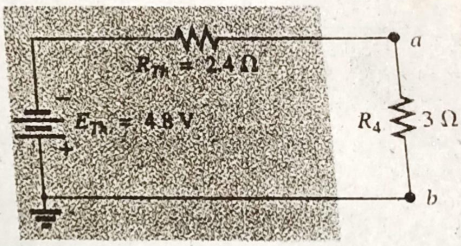


FIG. 9.42
Substituting the Thévenin equivalent circuit for the network external to the resistor R_4 in Fig. 9.37.

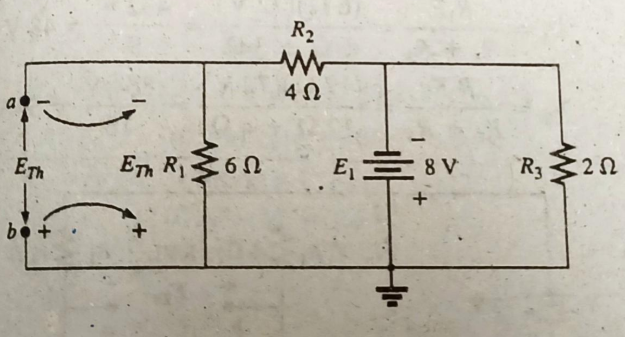


FIG. 9.40
Determining E_{Th} for the network in Fig. 9.38.

ep 5: See Fig. 9.42.

The importance of marking the terminals should be obvious from Example 9.8. Note that there is no requirement that the Thévenin voltage have the same polarity as the equivalent circuit originally introduced.

EXAMPLE 9.9 Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network in Fig. 9.43.

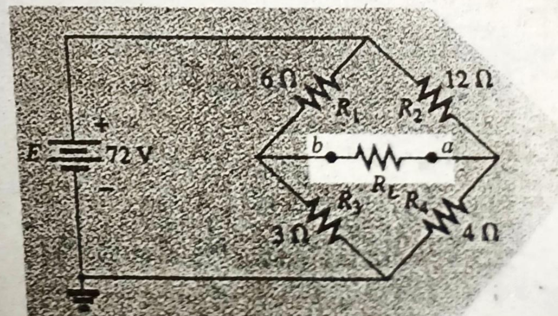


FIG. 9.43
Example 9.9.

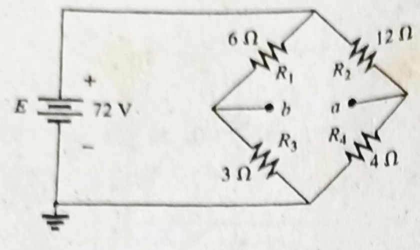


FIG. 9.44
Identifying the terminals of particular interest for the network in Fig. 9.43.

Solution:

Steps 1 and 2: See Fig. 9.44.

Step 3: See Fig. 9.45. In this case, the short-circuit replacement of the voltage source E provides a direct connection between c and c' in Fig. 9.45(a), permitting a "folding" of the network around the horizontal line of a-b to produce the configuration in Fig. 9.45(b).

$$\begin{aligned} R_{Th} = R_{a-b} &= R_1 \parallel R_3 + R_2 \parallel R_4 \\ &= 6\,\Omega \parallel 3\,\Omega + 4\,\Omega \parallel 12\,\Omega \\ &= 2\,\Omega + 3\,\Omega = \mathbf{5\,\Omega} \end{aligned}$$

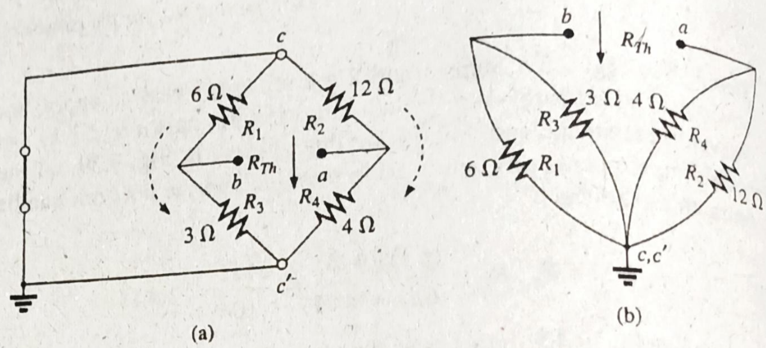


FIG. 9.45
Solving for R_{Th} for the network in Fig. 9.44.

Step 4: The circuit is redrawn in Fig. 9.46. The absence of a direct connection between a and b results in a network with three parallel branches. The voltages V_1 and V_2 can therefore be determined using the voltage divider rule:

$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6\,\Omega)(72\text{ V})}{6\,\Omega + 3\,\Omega} = \frac{432\text{ V}}{9} = 48\text{ V}$$

$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12\,\Omega)(72\text{ V})}{12\,\Omega + 4\,\Omega} = \frac{864\text{ V}}{16} = 54\text{ V}$$

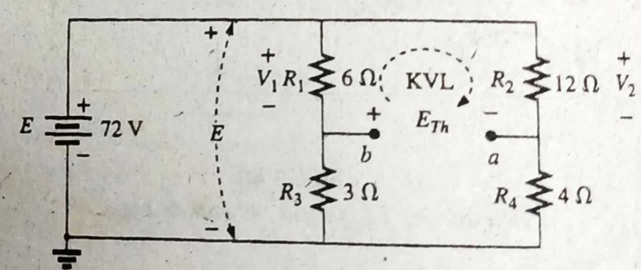


FIG. 9.46
Determining E_{Th} for the network in Fig. 9.44.

Assuming the polarity shown for E_{Th} and applying Kirchhoff's voltage law to the top loop in the clockwise direction results in

$$\Sigma_C\, V = +E_{Th} + V_1 - V_2 = 0$$

and

$$E_{Th} = V_2 - V_1 = 54\text{ V} - 48\text{ V} = 6\text{ V}$$

Step 5: See Fig. 9.47.

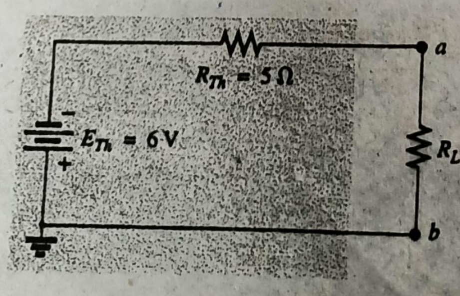


FIG. 9.47
Substituting the Thévenin equivalent circuit for the network external to the resistor R_L in Fig. 9.43.

Thévenin's theorem is not restricted to a single passive element, as shown in the preceding examples, but can be applied across sources, whole branches, portions of networks, or any circuit configuration as shown in the following example. It is also possible that you may have to use one of the methods previously described, such as mesh analysis or superposition, to find the Thévenin equivalent circuit.

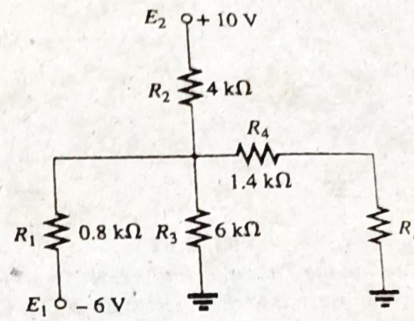


FIG. 9.48
Example 9.10.

EXAMPLE 9.10 (Two sources) Find the Thévenin circuit for the network within the shaded area of Fig. 9.48.

Solution:

Steps 1 and 2: See Fig. 9.49. The network is redrawn.

Step 3: See Fig. 9.50.

$$
\begin{aligned}
R_{Th} &= R_4 + R_1 \parallel R_2 \parallel R_3 \\
&= 1.4\ \text{k}\Omega + 0.8\ \text{k}\Omega \parallel 4\ \text{k}\Omega \parallel 6\ \text{k}\Omega \\
&= 1.4\ \text{k}\Omega + 0.8\ \text{k}\Omega \parallel 2.4\ \text{k}\Omega \\
&= 1.4\ \text{k}\Omega + 0.6\ \text{k}\Omega \\
&= \mathbf{2\ k\Omega}
\end{aligned}
$$

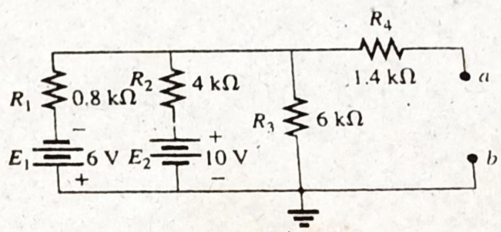


FIG. 9.49
Identifying the terminals of particular interest for the network in Fig. 9.48.

Step 4: Applying superposition, we will consider the effects of the voltage source E_1 first. Note Fig. 9.51. The open circuit requires that $V_4 = I_4R_4 = (0)R_4 = 0$ V, and

$$E'_{Th} = V_3$$

$$R'_T = R_2 \parallel R_3 = 4\ \text{k}\Omega \parallel 6\ \text{k}\Omega = 2.4\ \text{k}\Omega$$

Applying the voltage divider rule,

$$V_3 = \frac{R'_T E_1}{R'_T} = \frac{(2.4\ \text{k}\Omega)(6\ \text{V})}{2.4\ \text{k}\Omega + 0.8\ \text{k}\Omega} = \frac{14.4\ \text{V}}{3.2} = 4.5\ \text{V}$$

$$E'_{Th} = V_3 = 4.5\ \text{V}$$

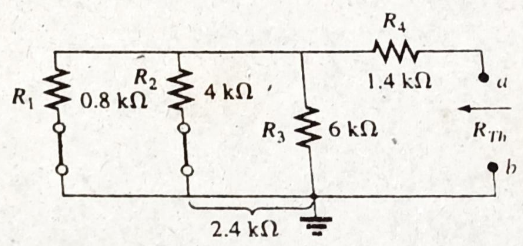


FIG. 9.50
Determining R_{Th} for the network in Fig. 9.49.

For the source E_2, the network in Fig. 9.52 results. Again, $V_4 = I_4R_4 = (0)R_4 = 0$ V, and

$$E''_{Th} = V_3$$

$$R'_T = R_1 \parallel R_3 = 0.8\ \text{k}\Omega \parallel 6\ \text{k}\Omega = 0.706\ \text{k}\Omega$$

$$V_3 = \frac{R'_T E_2}{R'_T + R_2} = \frac{(0.706\ \text{k}\Omega)(10\ \text{V})}{0.706\ \text{k}\Omega + 4\ \text{k}\Omega} = \frac{7.06\ \text{V}}{4.706} = 1.5\ \text{V}$$

$$E''_{Th} = V_3 = 1.5\ \text{V}$$

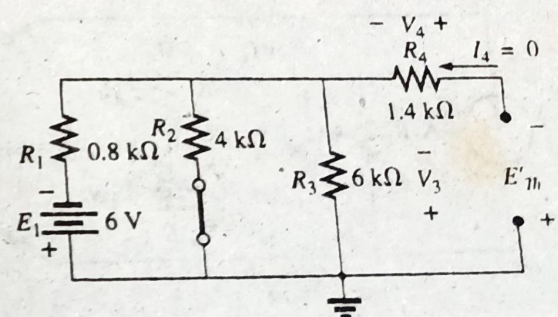


FIG. 9.51
Determining the contribution to E_{Th} from the source E_1 for the network in Fig. 9.49.

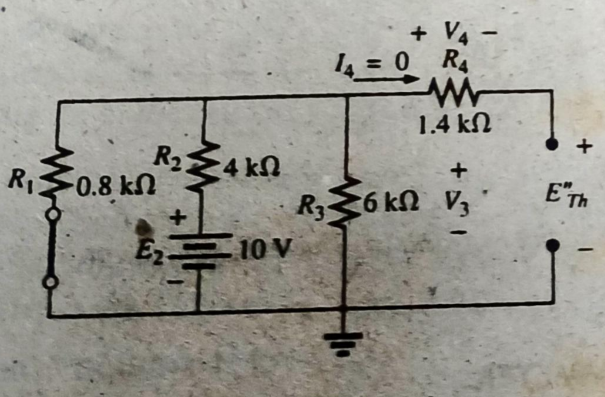


FIG. 9.52
Determining the contribution to E_{Th} from the source E_2 for the network in Fig. 9.49.

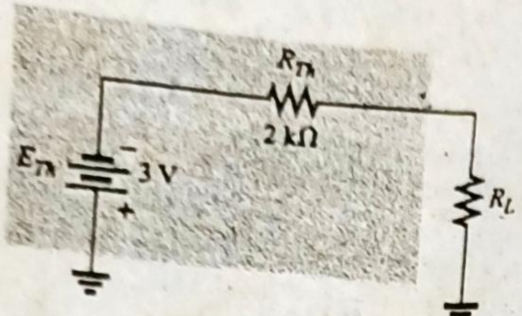


FIG. 9.53
Substituting the Thévenin equivalent circuit for the network external to the resistor R_L in Fig. 9.48.

Since E'_{Th} and E''_{Th} have opposite polarities,

$$\begin{aligned} E_{Th} &= E'_{Th} - E''_{Th} \\ &= 4.5\text{ V} - 1.5\text{ V} \\ &= \mathbf{3\text{ V}} \qquad \text{(polarity of } E'_{Th}\text{)} \end{aligned}$$

Step 5: See Fig. 9.53.

Experimental Procedures

Now that the analytical procedure has been described in detail and a sense for the Thévenin impedance and voltage established, it is time to in

marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$, the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.

I_N:

4. ***Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.***

Conclusion:

5. ***Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.***

The Norton and Thévenin equivalent circuits can also be found from each other by using the source transformation discussed earlier in this chapter and reproduced in Fig. 9.60.

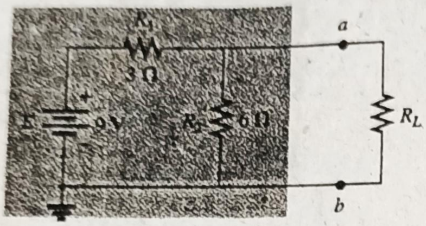


FIG. 9.61
Example 9.11.

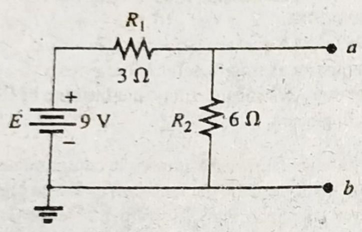


FIG. 9.62
Identifying the terminals of particular interest for the network in Fig. 9.61.

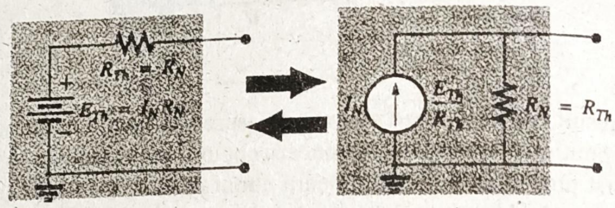


FIG. 9.60
Converting between Thévenin and Norton equivalent circuits.

EXAMPLE 9.11 Find the Norton equivalent circuit for the network in the shaded area in Fig. 9.61.

Solution:

Steps 1 and 2: See Fig. 9.62.

Step 3: See Fig. 9.63, and

$$R_N = R_1 \| R_2 = 3\ \Omega \| 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = 2\ \Omega$$

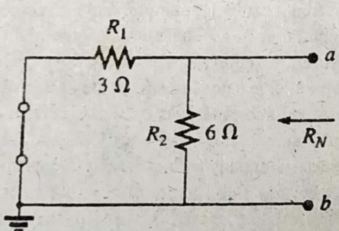


FIG. 9.63
Determining R_N for the network in Fig. 9.62.

Step 4: See Fig. 9.64, which clearly indicates that the short-circuit connection between terminals a and b is in parallel with R_2 and eliminates its effect. I_N is therefore the same as through R_1, and the full battery voltage appears across R_1 since

$$V_2 = I_2 R_2 = (0)6\ \Omega = 0\ \text{V}$$

Therefore,

$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = 3\ \text{A}$$

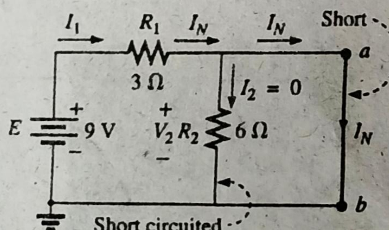


FIG. 9.64
Determining I_N for the network in Fig. 9.62.

Step 5: See Fig. 9.65. This circuit is the same as the first one considered in the development of Thévenin's theorem. A simple conversion indicates that the Thévenin circuits are, in fact, the same (Fig. 9.66).

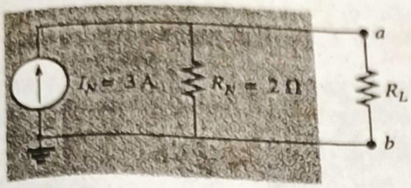


FIG. 9.65
Substituting the Norton equivalent circuit for the network external to the resistor R_L in Fig. 9.61.

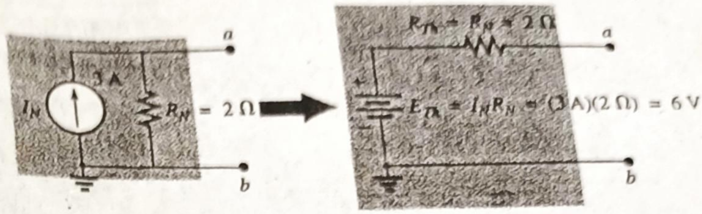


FIG. 9.66
Converting the Norton equivalent circuit in Fig. 9.65 to a Thévenin equivalent circuit.

EXAMPLE 9.12 Find the Norton equivalent circuit for the network external to the 9 Ω resistor in Fig. 9.67.

Solution:

Steps 1 and 2: See Fig. 9.68.

Step 3: See Fig. 9.69, and

$$R_N = R_1 + R_2 = 5\ \Omega + 4\ \Omega = \mathbf{9\ \Omega}$$

Step 4: As shown in Fig. 9.70, the Norton current is the same as the current through the 4 Ω resistor. Applying the current divider rule,

$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5\ \Omega)(10\ \text{A})}{5\ \Omega + 4\ \Omega} = \frac{50\ \text{A}}{9} = \mathbf{5.56\ A}$$

Step 5: See Fig. 9.71.

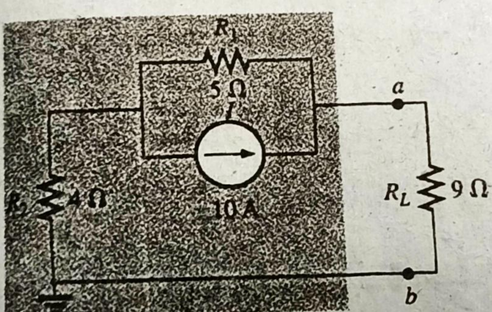


FIG. 9.67
Example 9.12.

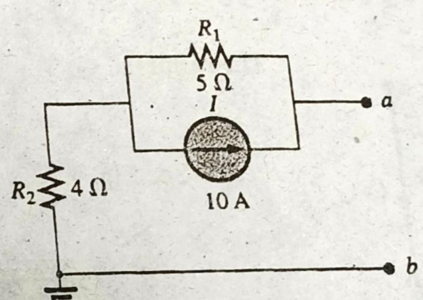


FIG. 9.68
Identifying the terminals of particular interest for the network in Fig. 9.67.

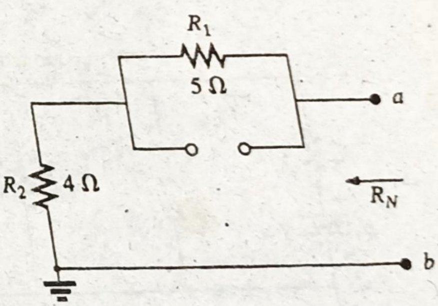


FIG. 9.69
Determining R_N for the network in Fig. 9.68.

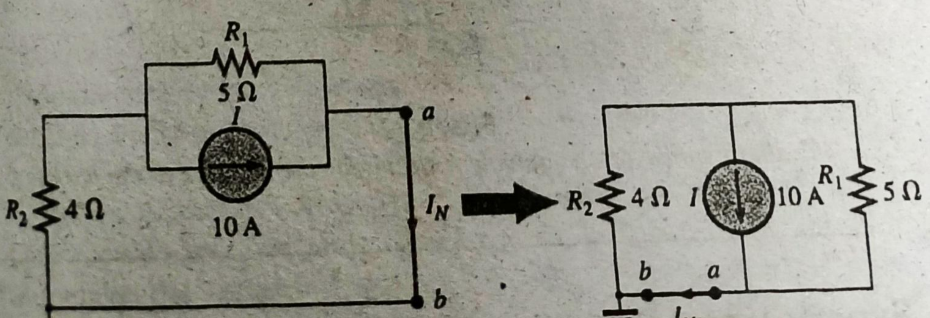


FIG. 9.70
Determining I_N for the network in Fig. 9.68.

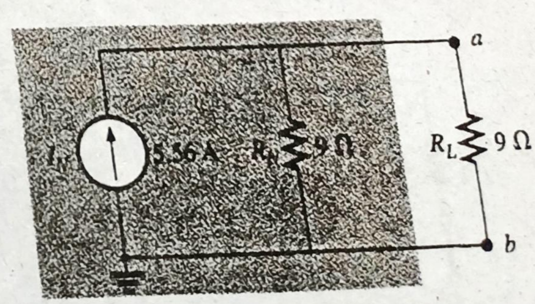


FIG. 9.71
Substituting the Norton equivalent circuit for the network external to the resistor R_L in Fig. 9.67.

EXAMPLE 9.13 (Two sources) Find the Norton equivalent circuit for the portion of the network to the left of *a-b* in Fig. 9.72.

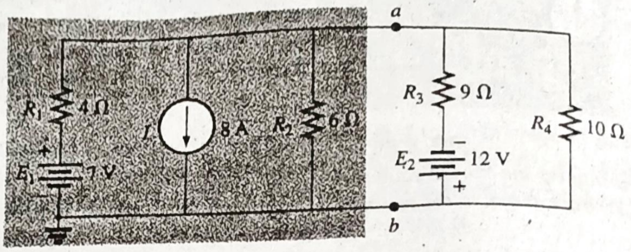


FIG. 9.72
Example 9.13.

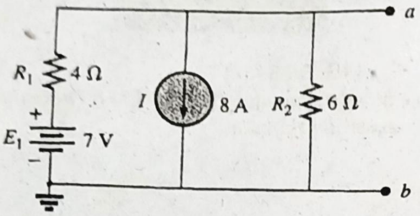


FIG. 9.73
Identifying the terminals of particular interest for the network in Fig. 9.72.

Solution:

Steps 1 and 2: See Fig. 9.73.

Step 3: See Fig. 9.74, and

$$R_N = R_1 \parallel R_2 = 4\ \Omega \parallel 6\ \Omega = \frac{(4\ \Omega)(6\ \Omega)}{4\ \Omega + 6\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

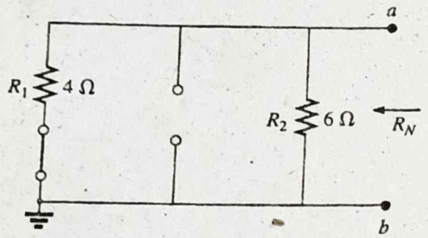


FIG. 9.74
Determining R_N for the network in Fig. 9.73.

Step 4: (Using superposition) For the 7 V battery (Fig. 9.75),

$$I'_N = \frac{E_1}{R_1} = \frac{7\ \text{V}}{4\ \Omega} = 1.75\ \text{A}$$

For the 8 A source (Fig. 9.76), we find that both R_1 and R_2 have been "short circuited" by the direct connection between *a* and *b*, and

$$I''_N = I = 8\ \text{A}$$

The result is

$$I_N = I''_N - I'_N = 8\ \text{A} - 1.75\ \text{A} = \mathbf{6.25\ A}$$

Step 5: See Fig. 9.77.

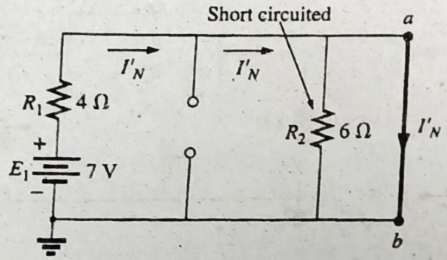


FIG. 9.75
Determining the contribution to I_N from the voltage source E_1.

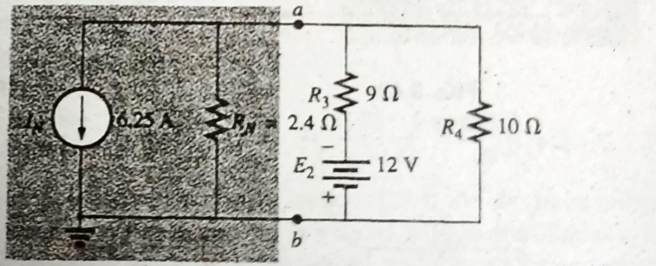


FIG. 9.77
Substituting the Norton equivalent circuit for the network to the left of terminals a-b in Fig. 9.72.

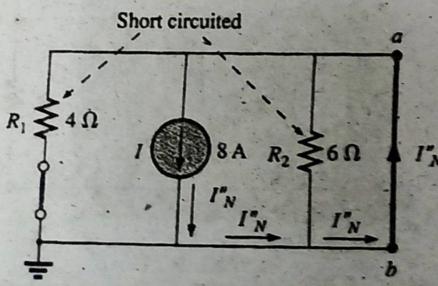


FIG. 9.76
Determining the contribution to I_N from the current source I.

Experimental Procedure

The Norton current is measured in the same way as described for the short-circuit current (I_{sc}) for the Thévenin network. Since the Norton and Thévenin resistances are the same, the same procedures can be followed as described for the Thévenin network.

In all of the above discussions, the effect of changing the load was discussed for a fixed Thévenin resistance. Looking at the situation from a different viewpoint,

if the load resistance is fixed and does not match the applied Thévenin equivalent resistance, then some effort should be made (if possible) to redesign the system so that the Thévenin equivalent resistance is closer to the fixed applied load.

In other words, if a designer faces a situation where the load resistance is fixed, he/she should investigate whether the supply section should be replaced or redesigned to create a closer match of resistance levels to produce higher levels of power to the load.

For the Norton equivalent circuit in Fig. 9.84, maximum power will be delivered to the load when

$$\boxed{R_L = R_N} \tag{9.5}$$

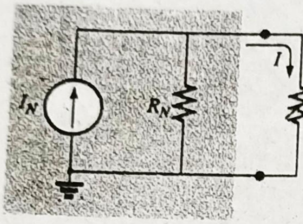


FIG. 9.84
Defining the conditions for maximum power to a load using the Norton equivalent circuit.

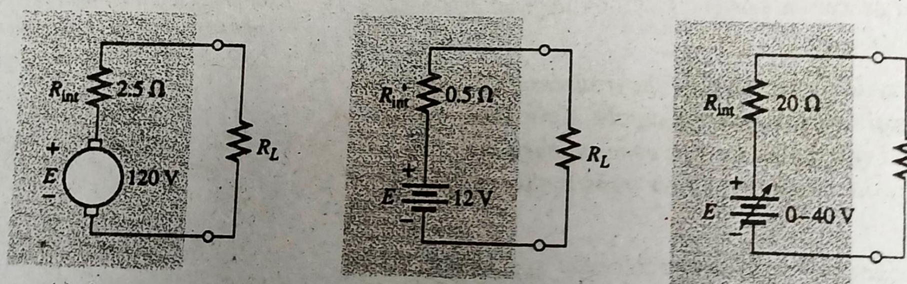
This result [Eq. (9.5)] will be used to its fullest advantage in the analysis of transistor networks, where the most frequently applied transistor circuit model uses a current source rather than a voltage source.

For the Norton circuit in Fig. 9.84,

$$\boxed{P_{L_{max}} = \frac{I_N^2 R_N}{4}} \quad \text{(W)} \tag{9.6}$$

EXAMPLE 9.14 A dc generator, battery, and laboratory supply are connected to resistive load R_L in Fig. 9.85.

a. For each, determine the value of R_L for maximum power transfer to R_L.
b. Under maximum power conditions, what are the current level and the power to the load for each configuration?
c. What is the efficiency of operation for each supply in part (b)?
d. If a load of 1 kΩ were applied to the laboratory supply, what would the power delivered to the load be? Compare your answer to the level of part (b). What is the level of efficiency?
e. For each supply, determine the value of R_L for 75% efficiency.



(a) dc generator (b) Battery (c) Laboratory supply

FIG. 9.85
Example 9.14.

Solutions:

a. For the dc generator,

$$R_L = R_{Th} = R_{int} = 2.5\ \Omega$$

For the 12 V car battery,

$$R_L = R_{Th} = R_{int} = \mathbf{0.05\ \Omega}$$

For the dc laboratory supply,

$$R_L = R_{Th} = R_{int} = \mathbf{20\ \Omega}$$

b. For the dc generator,

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{int}} = \frac{(120\ \text{V})^2}{4(2.5\ \Omega)} = \mathbf{1.44\ kW}$$

For the 12 V car battery,

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{int}} = \frac{(12\ \text{V})^2}{4(0.05\ \Omega)} = \mathbf{720\ W}$$

For the dc laboratory supply,

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{E^2}{4R_{int}} = \frac{(40\ \text{V})^2}{4(20\ \Omega)} = \mathbf{20\ W}$$

c. They are all operating under a 50% efficiency level because $R_L = R_{Th}$.

d. The power to the load is determined as follows:

$$I_L = \frac{E}{R_{int} + R_L} = \frac{40\ \text{V}}{20\ \Omega + 1000\ \Omega} = \frac{40\ \text{V}}{1020\ \Omega} = 39.22\ \text{mA}$$

and $\quad P_L = I_L^2 R_L = (39.22\ \text{mA})^2(1000\ \Omega) = \mathbf{1.54\ W}$

The power level is significantly less than the 20 W achieved in part (b). The efficiency level is

$$\eta\% = \frac{P_L}{P_s} \times 100\% = \frac{1.54\ \text{W}}{EI_s} \times 100\% = \frac{1.54\ \text{W}}{(40\ \text{V})(39.22\ \text{mA})} \times 100\%$$

$$= \frac{1.54\ \text{W}}{1.57\ \text{W}} \times 100\% = \mathbf{98.09\%}$$

which is markedly higher than achieved under maximum power conditions—albeit at the expense of the power level.

e. For the dc generator,

$$\eta = \frac{P_o}{P_s} = \frac{R_L}{R_{Th} + R_L} \qquad (\eta \text{ in decimal form})$$

and

$$\eta = \frac{R_L}{R_{Th} + R_L}$$

$$\eta(R_{Th} + R_L) = R_L$$

$$\eta R_{Th} + \eta R_L = R_L$$

$$R_L(1 - \eta) = \eta R_{Th}$$

and

$$\boxed{R_L = \frac{\eta R_{Th}}{1 - \eta}} \qquad (9.7)$$

$$R_L = \frac{0.75(2.5\ \Omega)}{1 - 0.75} = \mathbf{7.5\ \Omega}$$

For the battery,

$$R_L = \frac{0.75(0.05\ \Omega)}{1 - 0.75} = \mathbf{0.15\ \Omega}$$

For the laboratory supply,

$$R_L = \frac{0.75(20\ \Omega)}{1 - 0.75} = \mathbf{60\ \Omega}$$

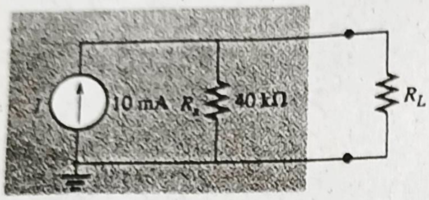


FIG. 9.86
Example 9.15.

EXAMPLE 9.15 The analysis of a transistor network resulted in the reduced equivalent in Fig. 9.86.

a. Find the load resistance that will result in maximum power transfer to the load, and find the maximum power delivered.
b. If the load were changed to 68 kΩ, would you expect a fairly high level of power transfer to the load based on the results of part (a)? What would the new power level be? Is your initial assumption verified?
c. If the load were changed to 8.2 kΩ, would you expect a fairly high level of power transfer to the load based on the results of part (a)? What would the new power level be? Is your initial assumption verified?

Solutions:

a. Replacing the current source by an open-circuit equivalent results in

$$R_{Th} = R_s = 40\ \text{k}\Omega$$

Restoring the current source and finding the open-circuit voltage at the output terminals results in

$$E_{Th} = V_{oc} = IR_s = (10\ \text{mA})(40\ \text{k}\Omega) = 400\ \text{V}$$

For maximum power transfer to the load,

$$R_L = R_{Th} = \mathbf{40\ k\Omega}$$

with a maximum power level of

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(400\ \text{V})^2}{4(40\ \text{k}\Omega)} = \mathbf{1\ W}$$

b. Yes, because the 68 kΩ load is greater (note Fig. 9.80) than the 40 kΩ load, but relatively close in magnitude.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{400\ \text{V}}{40\ \text{k}\Omega + 68\ \text{k}\Omega} = \frac{400}{108\ \text{k}\Omega} \cong 3.7\ \text{mA}$$

$$P_L = I_L^2 R_L = (3.7\ \text{mA})^2(68\ \text{k}\Omega \cong \mathbf{0.93\ W}$$

Yes, the power level of 0.93 W compared to the 1 W level of part (a) verifies the assumption.

c. No, 8.2 kΩ is quite a bit less (note Fig. 9.80) than the 40 kΩ value.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{400\ \text{V}}{40\ \text{k}\Omega + 8.2\ \text{k}\Omega} = \frac{400\ \text{V}}{48.2\ \text{k}\Omega} \cong 8.3\ \text{mA}$$

$$P_L = I_L^2 R_L = (8.3\ \text{mA})^2(8.2\ \text{k}\Omega) \cong \mathbf{0.57\ W}$$

Yes, the power level of 0.57 W compared to the 1 W level of part (a) verifies the assumption.

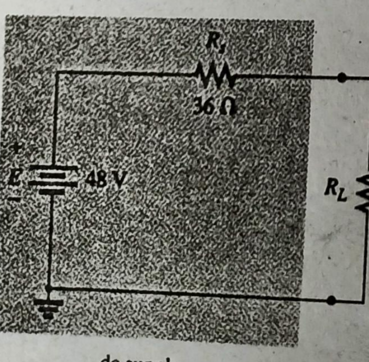


FIG. 9.87
dc supply with a fixed 16 Ω load (Example 9.16).

EXAMPLE 9.16 In Fig. 9.87, a fixed load of 16 Ω is applied to a 48 V supply with an internal resistance of 36 Ω.

a. For the conditions in Fig. 9.87, what is the power delivered to the load and lost to the internal resistance of the supply?
b. If the designer has some control over the internal resistance level of the supply, what value should he/she make it for maximum power to the load? What is the maximum power to the load? How does it compare to the level obtained in part (a)?
c. Without making a single calculation, if the designer could change the internal resistance to 22 Ω or 8.2 Ω, which value would result in more power to the load? Verify your conclusion by calculating the power to the load for each value.

Solutions:

a.
$$I_L = \frac{E}{R_s + R_L} = \frac{48\text{ V}}{36\ \Omega + 16\ \Omega} = \frac{48\text{ V}}{52\ \Omega} = 923.1\text{ mA}$$

$$P_{R_s} = I_L^2 R_s = (923.1\text{ mA})^2(36\ \Omega) = \mathbf{30.68\text{ W}}$$
$$P_L = I_L^2 R_L = (923.1\text{ mA})^2(16\ \Omega) = \mathbf{13.63\text{ W}}$$

b. Be careful here. The quick response is to make the source resistance R_s equal to the load resistance to satisfy the criteria of the maximum power transfer theorem. However, this is a totally different type of problem from what was examined earlier in this section. If the load is fixed, the smaller the source resistance R_s, the more applied voltage will reach the load and the less will be lost in the internal series resistor. In fact, the source resistance should be made as small as possible. If zero ohms were possible for R_s, the voltage across the load would be the full supply voltage, and the power delivered to the load would equal

$$P_L = \frac{V_L^2}{R_L} = \frac{(48\text{ V})^2}{16\ \Omega} = \mathbf{144\text{ W}}$$

which is more than 10 times the value with a source resistance of 36 Ω.

c. Again, forget the impact in Fig. 9.80: The smaller the source resistance, the greater the power to the fixed 16 Ω load. Therefore, the 8.2 Ω resistance level results in a higher power transfer to the load than the 22 Ω resistor.

For $R_s = 8.2\ \Omega$:

$$I_L = \frac{E}{R_s + R_L} = \frac{48\text{ V}}{8.2\ \Omega + 16\ \Omega} = \frac{48\text{ V}}{24.2\ \Omega} = 1.983\text{ A}$$

and $$P_L = I_L^2 R_L = (1.983\text{ A})^2(16\ \Omega) \cong \mathbf{62.92\text{ W}}$$

For $R_s = 22\ \Omega$:

$$I_L = \frac{E}{R_s + R_L} = \frac{48\text{ V}}{22\ \Omega + 16\ \Omega} = \frac{48\text{ V}}{38\ \Omega} = 1.263\text{ A}$$

and $$P_L = I_L^2 R_L = (1.263\text{ A})^2(16\ \Omega) \cong \mathbf{25.52\text{ W}}$$

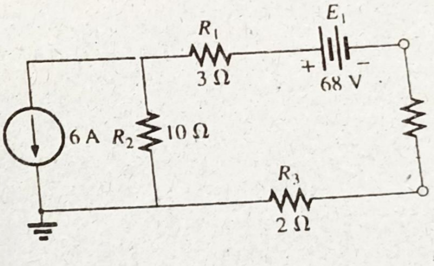


FIG. 9.88
Example 9.17.

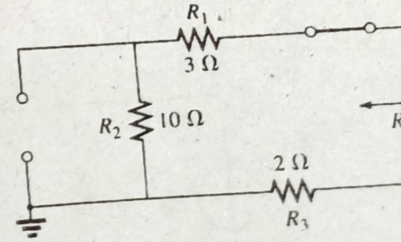


FIG. 9.89
Determining R_{Th} for the network external to R_L in Fig. 9.88.

EXAMPLE 9.17 Given the network in Fig. 9.88, find the value of R_L for maximum power to the load, and find the maximum power to the load.

Solution: The Thévenin resistance is determined from Fig. 9.89.

$$R_{Th} = R_1 + R_2 + R_3 = 3\ \Omega + 10\ \Omega + 2\ \Omega = 15\ \Omega$$

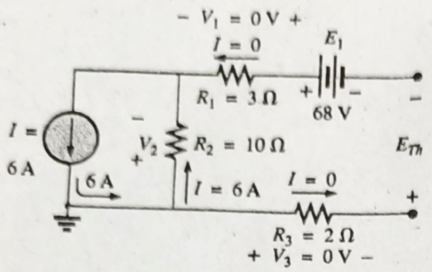


FIG. 9.90
Determining E_{Th} for the network external to resistor R_L in Fig. 9.88.

so that

$$R_L = R_{Th} = \mathbf{15\ \Omega}$$

The Thévenin voltage is determined using Fig. 9.90, where

$$V_1 = V_3 = 0\text{ V} \quad \text{and} \quad V_2 = I_2R_2 = IR_2 = (6\text{ A})(10\ \Omega) = 60\text{ V}$$

Applying Kirchhoff's voltage law:

$$-V_2 - E + E_{Th} = 0$$

and

$$E_{Th} = V_2 + E = 60\text{ V} + 68\text{ V} = \mathbf{128\ V}$$

with the maximum power equal to

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\text{ V})^2}{4(15\text{ k}\Omega)} = \mathbf{273.07\ W}$$

9.6 MILLMAN'S THEOREM

Through the application of **Millman's theorem,** any number of parallel voltage sources can be reduced to one. In Fig. 9.91, for example, the three voltage sources can be reduced to one. This permits finding the current through or voltage across R_L without having to apply a method such as mesh analysis, nodal analysis, superposition, and so on. The theorem can best be described by applying it to the network in Fig. 9.91. Basically, three steps are included in its application.

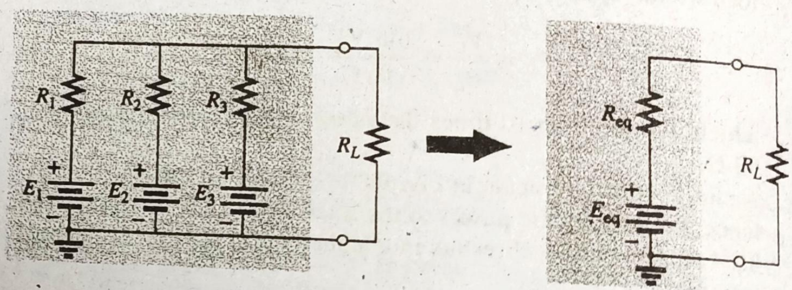


FIG. 9.91
Demonstrating the effect of applying Millman's theorem.

Step 1: Convert all voltage sources to current sources as outlined in Section 8.3. This is performed in Fig. 9.92 for the network in Fig. 9.91.

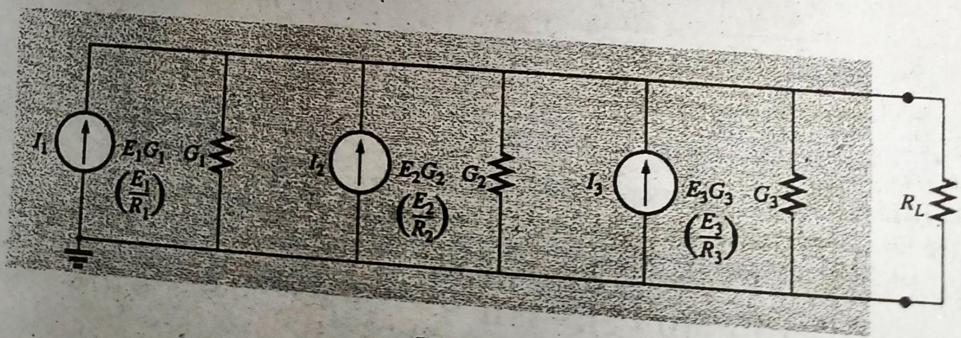


FIG. 9.92
Converting all the sources in Fig. 9.91 to current sources.

Step 2: Combine parallel current sources as described in Section 8.4. The resulting network is shown in Fig. 9.93, where

$$I_T = I_1 + I_2 + I_3 \quad \text{and} \quad G_T = G_1 + G_2 + G_3$$

Step 3: Convert the resulting current source to a voltage source, and the desired single-source network is obtained, as shown in Fig. 9.94.

In general, Millman's theorem states that for any number of parallel voltage sources,

$$E_{eq} = \frac{I_T}{G_T} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm \cdots \pm I_N}{G_1 + G_2 + G_3 + \cdots + G_N}$$

or

$$\boxed{E_{eq} = \frac{\pm E_1G_1 \pm E_2G_2 \pm E_3G_3 \pm \cdots \pm E_NG_N}{G_1 + G_2 + G_3 + \cdots + G_N}} \tag{9.8}$$

The plus-and-minus signs appear in Eq. (9.8) to include those cases where the sources may not be supplying energy in the same direction. (Note Example 9.18.)

The equivalent resistance is

$$\boxed{R_{eq} = \frac{1}{G_T} = \frac{1}{G_1 + G_2 + G_3 + \cdots + G_N}} \tag{9.9}$$

In terms of the resistance values,

$$\boxed{E_{eq} = \frac{\pm \dfrac{E_1}{R_1} \pm \dfrac{E_2}{R_2} \pm \dfrac{E_3}{R_3} \pm \cdots \pm \dfrac{E_N}{R_N}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}} \tag{9.10}$$

and

$$\boxed{R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}} \tag{9.11}$$

Because of the relatively few direct steps required, you may find it easier to apply each step rather than memorizing and employing Eqs. (9.8) through (9.11).

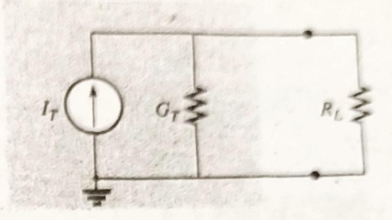


FIG. 9.93
Reducing all the current sources in Fig. 9.92 to a single current source.

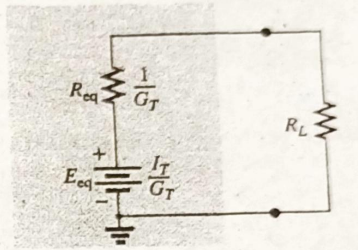


FIG. 9.94
Converting the current source in Fig. 9.93 to a voltage source.

EXAMPLE 9.18 Using Millman's theorem, find the current through and voltage across the resistor R_L in Fig. 9.95.

Solution: By Eq. (9.10),

$$E_{eq} = \frac{+\dfrac{E_1}{R_1} - \dfrac{E_2}{R_2} + \dfrac{E_3}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

The minus sign is used for E_2/R_2 because that supply has the opposite polarity of the other two. The chosen reference direction is therefore that of E_1 and E_3. The total conductance is unaffected by the direction, and

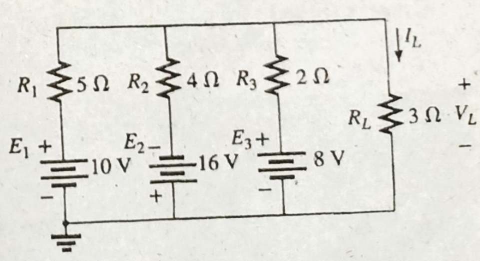


FIG. 9.95
Example 9.18.

$$E_{eq} = \frac{+\dfrac{10\text{ V}}{5\,\Omega} - \dfrac{16\text{ V}}{4\,\Omega} + \dfrac{8\text{ V}}{2\,\Omega}}{\dfrac{1}{5\,\Omega} + \dfrac{1}{4\,\Omega} + \dfrac{1}{2\,\Omega}} = \frac{2\text{ A} - 4\text{ A} + 4\text{ A}}{0.2\text{ S} + 0.25\text{ S} + 0.5\text{ S}}$$

$$= \frac{2\text{ A}}{0.95\text{ S}} = \mathbf{2.11\text{ V}}$$

with
$$R_{eq} = \frac{1}{\dfrac{1}{5\,\Omega} + \dfrac{1}{4\,\Omega} + \dfrac{1}{2\,\Omega}} = \frac{1}{0.95\text{ S}} = \mathbf{1.05\,\Omega}$$

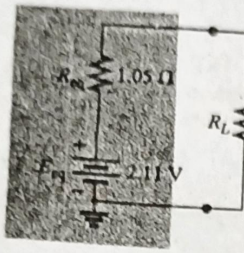


FIG. 9.96
The result of applying Millman's theorem to the network in Fig. 9.95.

The resultant source is shown in Fig. 9.96, and

$$I_L = \frac{2.11\text{ V}}{1.05\,\Omega + 3\,\Omega} = \frac{2.11\text{ V}}{4.05\,\Omega} = \mathbf{0.52\text{ A}}$$

with
$$V_L = I_L R_L = (0.52\text{ A})(3\,\Omega) = \mathbf{1.56\text{ V}}$$

EXAMPLE 9.19 Let us now consider the type of problem encountered in the introduction to mesh and nodal analysis in Chapter 8. Mesh analysis was applied to the network of Fig. 9.97 (Example 8.12). Let us now use Millman's theorem to find the current through the 2 Ω resistor and compare the results.

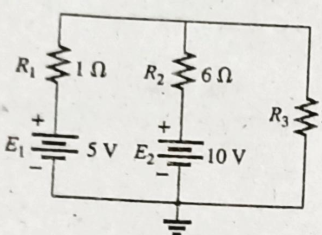


FIG. 9.97
Example 9.19.

Solutions:

a. Let us first apply each step and, in the (b) solution, Eq. (9.10). Converting sources yields Fig. 9.98. Combining sources and parallel conductance branches (Fig. 9.99) yields

$$I_T = I_1 + I_2 = 5\text{ A} + \frac{5}{3}\text{ A} = \frac{15}{3}\text{ A} + \frac{5}{3}\text{ A} = \frac{20}{3}\text{ A}$$

$$G_T = G_1 + G_2 = 1\text{ S} + \frac{1}{6}\text{ S} = \frac{6}{6}\text{ S} + \frac{1}{6}\text{ S} = \frac{7}{6}\text{ S}$$

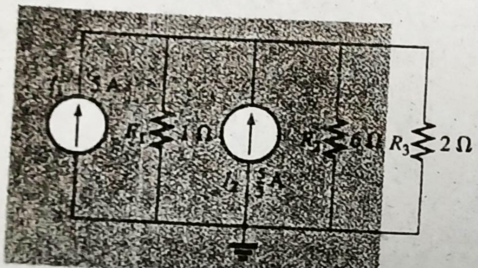


FIG. 9.98
Converting the sources in Fig. 9.97 to current sources.

Converting the current source to a voltage source (Fig. 9.100), we obtain

$$E_{eq} = \frac{I_T}{G_T} = \frac{\dfrac{20}{3}\text{ A}}{\dfrac{7}{6}\text{ S}} = \frac{(6)(20)}{(3)(7)}\text{ V} = \mathbf{\frac{40}{7}\text{ V}}$$

and
$$R_{eq} = \frac{1}{G_T} = \frac{1}{\dfrac{7}{6}\text{ S}} = \mathbf{\frac{6}{7}\,\Omega}$$

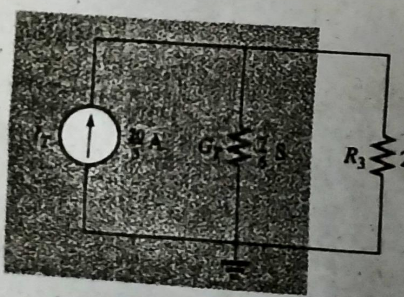


FIG. 9.99
Reducing the current sources in Fig. 9.98 to a single source.

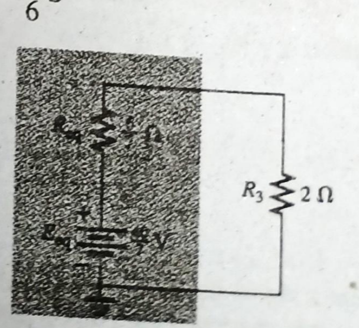


FIG. 9.100
Converting the current source in Fig. 9.99 to a voltage source.

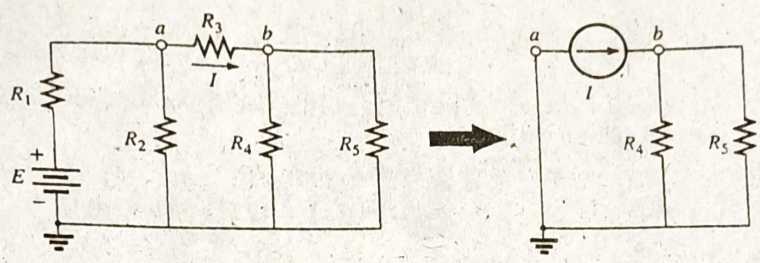


FIG. 9.105
Demonstrating the effect of knowing a current at some point in a complex network.

9.8 RECIPROCITY THEOREM

The **reciprocity theorem** is applicable only to single-source networks. It is, therefore, not a theorem used in the analysis of multisource networks described thus far. The theorem states the following:

The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in current. The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.

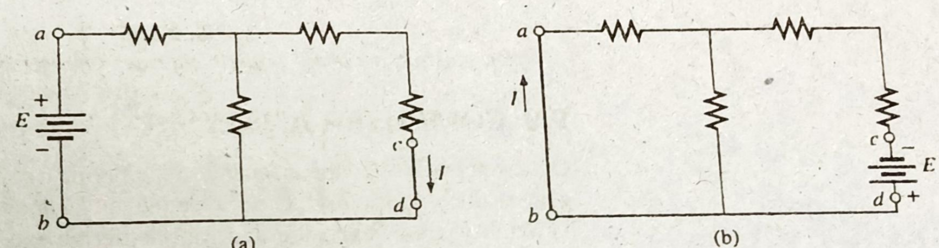


FIG. 9.106
Demonstrating the impact of the reciprocity theorem.

In the representative network in Fig. 9.106(a), the current I due to the voltage source E was determined. If the position of each is interchanged as shown in Fig. 9.106(b), the current I will be the same value as indicated. To demonstrate the validity of this statement and the theorem, consider the network in Fig. 9.107, in which values for the elements of Fig. 9.106(a) have been assigned.

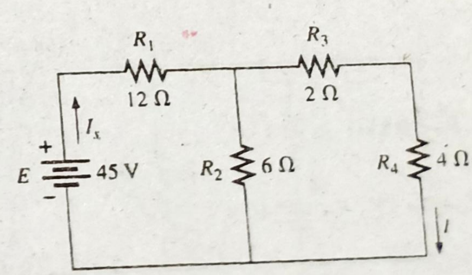


FIG. 9.107
Finding the current I due to a source E.

The total resistance is

$$R_T = R_1 + R_2 \| (R_3 + R_4) = 12\,\Omega + 6\,\Omega \| (2\,\Omega + 4\,\Omega)$$
$$= 12\,\Omega + 6\,\Omega \| 6\,\Omega = 12\,\Omega + 3\,\Omega = 15\,\Omega$$

and

$$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{15\,\Omega} = 3\text{ A}$$

$$I = \frac{3\text{ A}}{2} = \mathbf{1.5\ A}$$

with

For the network in Fig. 9.108, which corresponds to that in Fig. 9.106(b), we find

$$R_T = R_4 + R_3 + R_1 \parallel R_2$$
$$= 4\ \Omega + 2\ \Omega + 12\ \Omega \parallel 6\ \Omega = 10\ \Omega$$

and

$$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{10\ \Omega} = 4.5\text{ A},$$

so that

$$I = \frac{(6\ \Omega)(4.5\text{ A})}{12\ \Omega + 6\ \Omega} = \frac{4.5\text{ A}}{3} = \mathbf{1.5\ A}$$

which agrees with the above.

The uniqueness and power of this theorem can best be demonstrated by considering a complex, single-source network such as the one shown in Fig. 9.109.

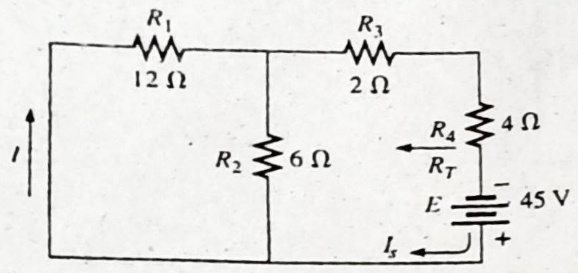


FIG. 9.108
Interchanging the location of E and I of Fig. 9.107 to demonstrate the validity of the reciprocity theorem.

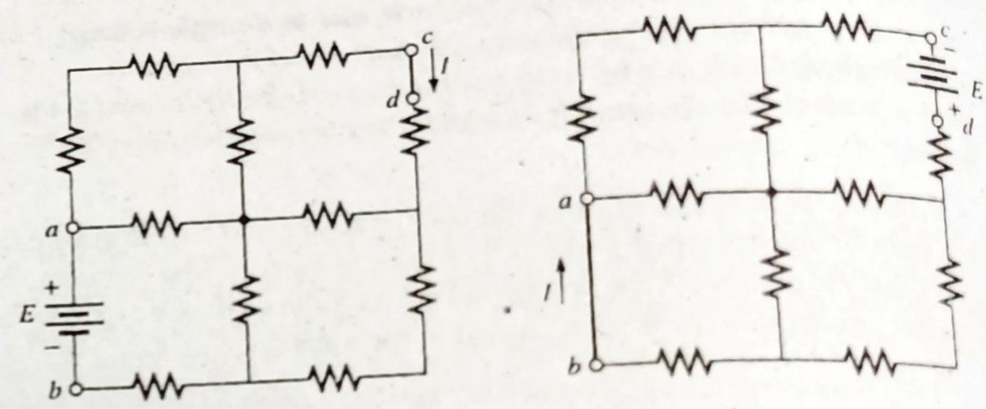


FIG. 9.109
Demonstrating the power and uniqueness of the reciprocity theorem.