Kirchhoff's Current Law (KCL)

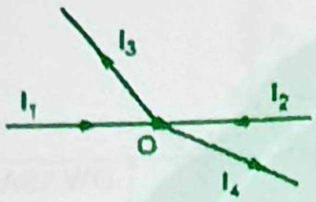


Fig. 1.37 Junction point

The total current flowing towards a junction point is equal to the total current flowing away from that junction point.

Another way to state the law is,

The algebraic sum of all the current meeting at a junction point is always zero.

The word algebraic means considering the signs of various currents.

$$\sum I \text{ at junction point} = 0$$

Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.

e.g. Refer to Fig. 1.37, currents I_1 and I_2 are positive while I_3 and I_4 are negative.

Applying KCL, $\sum I$ at junction O $= 0$

$$I_1 + I_2 - I_3 - I_4 = 0 \text{ i.e. } I_1 + I_2 = I_3 + I_4$$

Kirchhoff's Voltage Law (KVL)

"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f. s in the path"

In other words, "The algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

$$\text{Around a closed path } \sum V = 0$$

Network

Any arrangement of the various electrical energy sources along with the different circuit elements is called an electrical network.

Any individual circuit element with two terminals which can be connected to other circuit element is called a network element. Network elements can be either active elements or passive elements.

Active elements are the elements which supply power or energy to the network. Voltage source and current source are the examples of active elements.

assive elements are the elements which either store energy or dissipate energy in the form of heat. Resistor, nductor and capacitor are the three basic passive elements. Inductors and capacitors can store energy and resistors issipate energy in the form of heat.

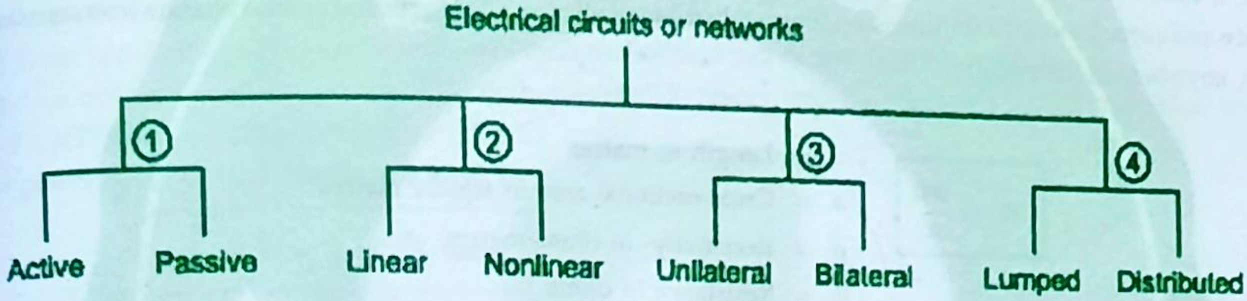
Branch

A part of the network which connects the various points of the network with one another is called a branch.

Mesh (or Loop)

Moth (or Loop) is a set of branches forming a closed path in a network in such a way that if one branch is removed hen refraining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes, without travelling hrough any node twice.

The Classification of network can be shown as



Linear Network:

A circuit or network whose parameters i.e. elements like resistances, inductances and capacitances are always constant irrespective of the change in time, voltage, temperature etc. is known as linear network. The Ohm's law can be applied to such network. The mathematical equations of such network can be obtained by using the law of superposition. The response of the various network elements is linear with respect to the excitation applied to them.

Non-linear Network:

A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as non-linear network. The Ohm's law may not be applied to such network. Such network does not follow the law of superposition. The response of the various elements is not linear with respect to their excitation. The best example is a circuit consisting of a diode where diode current does not vary linearly with the voltage applied to it.

Bilateral Network:

A circuit whose characteristics, behavior is same **irrespective of the direction of current** through various elements of it, is called bilateral network. Network consisting only resistances is good example of bilateral network.

Unilateral Network:

A circuit whose operation, behavior is dependent on the direction of the current through various elements is called unilateral network. Circuit consisting diodes, which allows flow of current only in one direction is good example of unilateral circuit.

Active Network:

A circuit which contains at least one **source of energy** is called active. An energy source may be a voltage or current source.

Passive Network:

A circuit which contains no energy source is called passive circuit.

Lumped Network:

A network in which all the **network elements are physically separable** is known as lumped network. Most of the electric networks are lumped in nature, which consist elements like R, L, C, voltage source etc.

Distributed Network:

A network in which the circuit elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called distributed network. The best example of such a network is a transmission line where resistance, inductance and capacitance of a transmission line are distributed all along its length and cannot be shown as separate elements, anywhere in the circuit.

$$R = \frac{\rho l}{a}$$

l = Length in metres

a = Cross-sectional area in square metres

ρ = Resistivity in ohms-metres

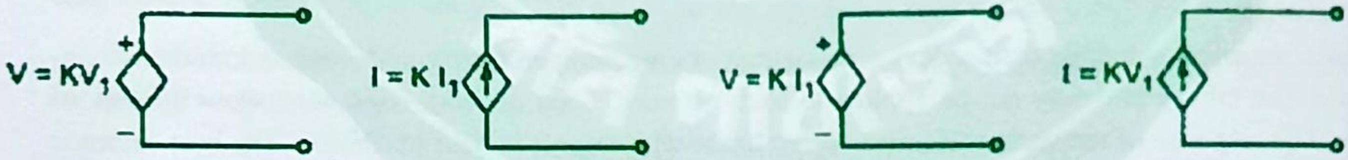
R = Resistance in ohms

4.186 Joules = 1 Calorie

1 Joule = 0.24 Calorie

Dependent Sources

Dependent sources are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the Fig. and further classified as,



(a) (b) (c) (d)

i) Voltage Dependent Voltage Source: It produces a voltage as a function of voltages elsewhere in the given circuit. This is called VDVS. It is shown in the Fig. (a).

ii) Current Dependent Current Source: It produces a current as a function of currents elsewhere in the given circuit. This is called CDCS. It is shown in the Fig. (b).

iii) Current Dependent Voltage Source: It produces a voltage as a function of current elsewhere in the given circuit. This is called CDVS. It is shown in the Fig. (c).

(iv) Voltage Dependent Current Source: It produces a current as a function of voltage elsewhere in the given circuit. This is called VDCS. It is shown in the Fig. (d).

The current flowing through the ...

10. Thevenin's Theorem

Fig. 3.8 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. The network in the box may consist of any number of resistors and e.m.f. sources connected in any manner. But according to Thevenin, the entire circuit behind terminals *A* and *B* can be replaced by a single source of e.m.f. E_{Th} (called Thevenin voltage) in series with a single resistance R_{Th} (called Thevenin resistance) as shown in Fig. 3.8 (*ii*). The values of E_{Th} and R_{Th} are determined as mentioned in Thevenin's theorem. Once *Thevenin's equivalent circuit* is obtained [See Fig. 3.8 (*ii*)], then current *I* through any load resistance R_L connected across *AB* is given by ;

$$I = \frac{E_{Th}}{R_{Th} + R_L}$$

In mesh *CBDC*, the voltage drop across R_3 is $E_2 - V_B$.

$$I_2 = \frac{E_2 - V_B}{R_3}$$

∴

Also current $I_3 = V_B/R_2$. Putting the values of I_1, I_2 and I_3 in eq. (*i*), we get,

$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_2} \quad ...(ii)$$

All quantities except V_B are known. Hence V_B can be found out. Once V_B is known, all branch currents can be calculated. It may be seen that nodal analysis requires only one equation [eq. (*ii*)] for determining the branch currents in this circuit. However, Kirchhoff's or Maxwell's solution would have needed two equations.

9. Superposition Theorem

Superposition is a general principle that allows us to determine the effect of several energy sources (voltage and current sources) acting simultaneously in a circuit by considering the effect of each source acting alone, and then combining (superposing) these effects. This theorem as applied to d.c. circuits may be stated as under :

In a linear, bilateral d.c. network containing more than one energy source, the resultant potential difference across or current through any element is equal to the algebraic sum of potential differences or currents for that element produced by each source acting alone with all other independent ideal voltage sources replaced by short circuits and all other independent ideal current sources replaced by open circuits (non-ideal sources are replaced by their internal resistances).

Procedure. The procedure for using this theorem to solve d.c. networks is as under :

(*i*) Select one source in the circuit and replace all other ideal voltage sources by short circuits and ideal current sources by open circuits.

(*ii*) Determine the voltage across or current through the desired element/branch due to single source selected in step (*i*).

(*iii*) Repeat the above two steps for each of the remaining sources.

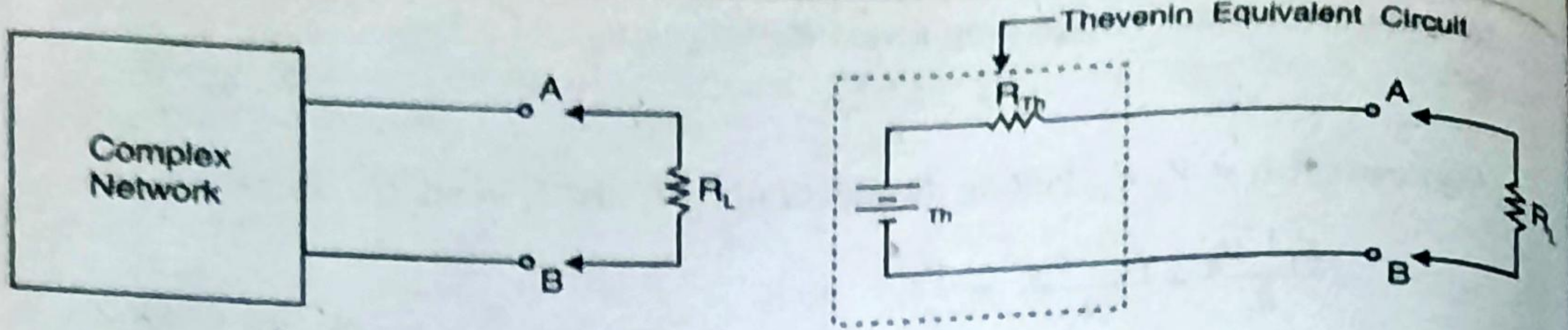
(*iv*) *Algebraically* add all the voltages across or currents through the element/branch under consideration. The sum is the actual voltage across or current through that element/branch when all the sources are acting simultaneously.

Note. This theorem is called *superposition* because we superpose or algebraically add the components (currents or voltages) due to each independent source acting alone to obtain the total current in or voltage across a circuit element.

10. Thevenin's Theorem

Fig. 3.8 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. The network in the box may consist of any number of resistors and e.m.f. sources connected in any manner. But according to Thevenin, the entire circuit behind terminals *A* and *B* can be replaced by a single source of e.m.f. E_{Th} (called Thevenin voltage) in series with a single resistance R_{Th} (called Thevenin resistance) as shown in Fig. 3.8 (*ii*). The values of E_{Th} and R_{Th} are determined as mentioned in Thevenin's theorem. Once *Thevenin's equivalent circuit* is obtained [See Fig. 3.8 (*ii*)], then current *I* through any load resistance R_L connected across *AB* is given by ;

$$I = \frac{E_{Th}}{R_{Th} + R_L}$$



(i) (ii)

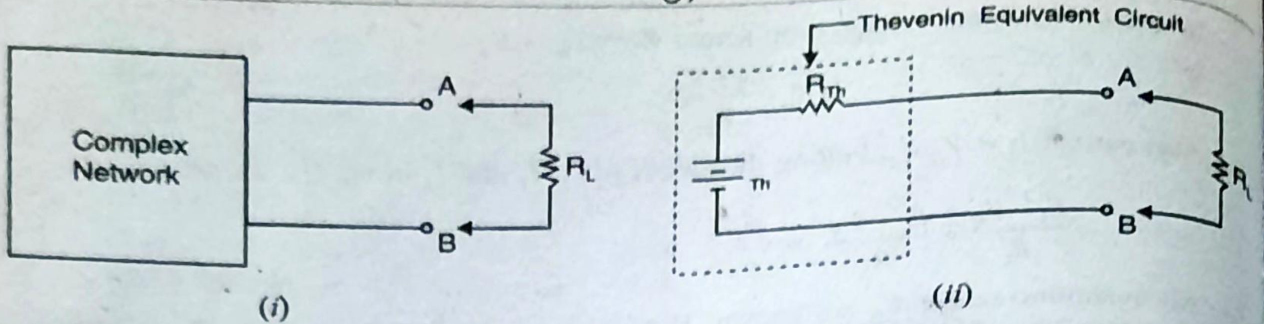
Fig. 3.8

Thevenin's theorem as applied to d.c. circuits is stated below :

Any linear, bilateral network having terminals A and B can be replaced by a single source of e.m.f. E_{Th} *in series with a single resistance* R_{Th}.

(i) *The e.m.f.* E_{Th} *is the voltage obtained across terminals A and B with load, if any removed i.e. it is open-circuited voltage between terminals A and B.*

(ii) *The resistance* R_{Th} *is the resistance of the network measured between terminals A and B with load removed and sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.*



(i) (ii)

Fig. 3.8

Thevenin's theorem as applied to d.c. circuits is stated below :

Any linear, bilateral network having terminals A and B can be replaced by a single source of e.m.f. E_{Th} in series with a single resistance R_{Th}.

(i) *The e.m.f. E_{Th} is the voltage obtained across terminals A and B with load, if any removed i.e. it is open-circuited voltage between terminals A and B.*

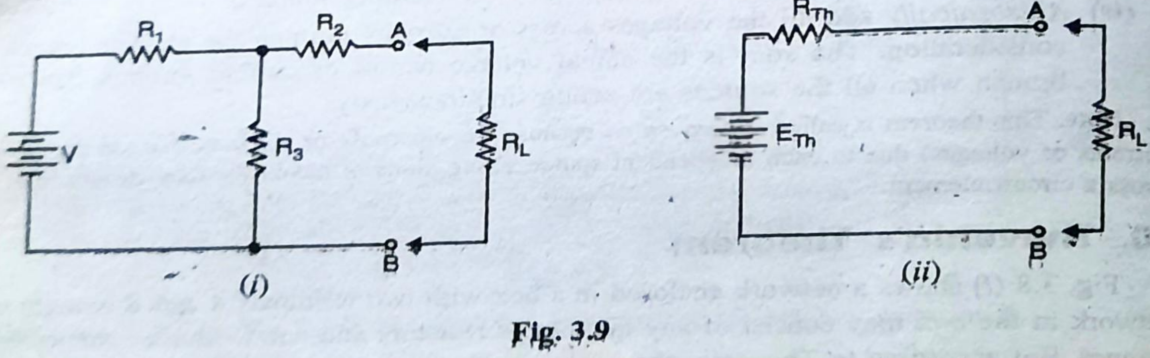
(ii) *The resistance R_{Th} is the resistance of the network measured between terminals A and B with load removed and sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.*

Illustration. Consider the circuit shown in Fig. 3.9 (i). As far as the circuit behind terminals AB is concerned, it can be replaced by a single source of e.m.f. E_{Th} in series with a single resistance R_{Th} as shown in Fig. 3.9 (ii). The e.m.f. E_{Th} is the voltage across terminals AB with R_L removed. With R_L disconnected, there is no current in R_2 and E_{Th} will be voltage appearing across R_3.

$$\therefore \quad E_{Th} = \text{Voltage across } R_3$$

$$= \text{Current through } R_3 \times \text{Resistance } R_3$$

$$= \frac{V}{R_1 + R_3} \times R_3$$



(i) (ii)

Fig. 3.9

To find R_{Th}, remove the load R_L and replace the battery by a short-circuit because its internal resistance is assumed zero. Then resistance measured between A and B is equal to R_{Th}. Obviously, looking back into the terminals AB, R_1 and R_3 are in parallel and this parallel combination is in series with R_2.

$$\therefore \quad R_{Th} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

11. Norton's Theorem

Fig. 3.10 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. The network in the box may contain any number of resistors and e.m.f. sources connected in any manner. But according to Norton, the entire circuit behind *AB* can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. 3.10 (*ii*). The resistance R_N is the same as Thevenin resistance R_{Th}. The value of I_N is determined as mentioned in Norton's theorem. Once Norton's equivalent circuit is determined [See Fig. 3.10 (*ii*)], then current through any load R_L connected across *AB* can be readily obtained.

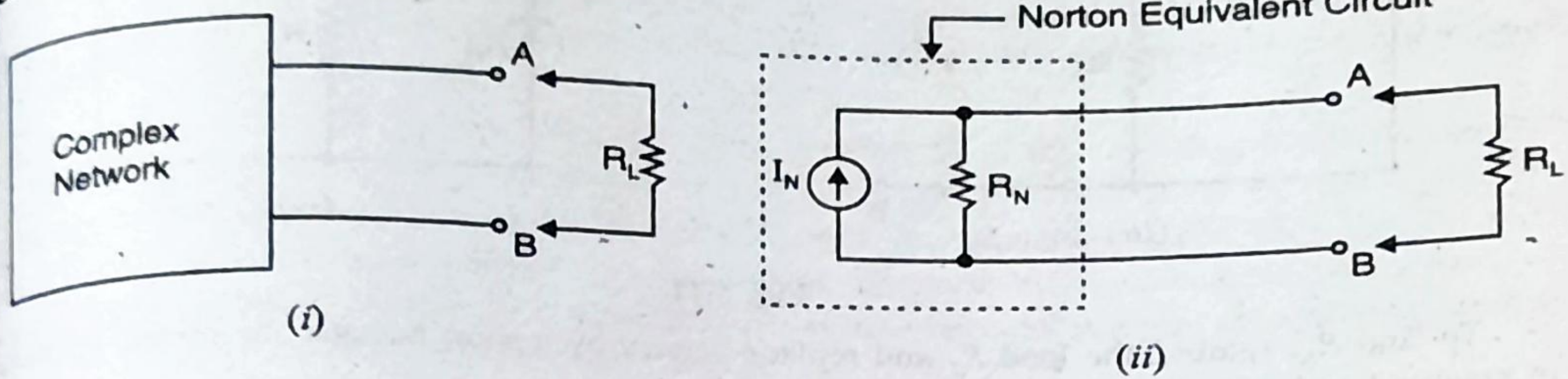


Fig. 3.10

Hence, Norton's theorem as applied to d.c. circuits may be stated as under :

Any linear, bilateral network having two terminals A and B can be replaced by a current source of current output I_N in parallel with a resistance R_N.

(*i*) *The output I_N of the current source is equal to the current that would flow through AB when A and B are short-circuited.*

(*ii*) *The resistance R_N is the resistance of the network measured between A and B with load removed and the sources of e.m.f./current replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.*

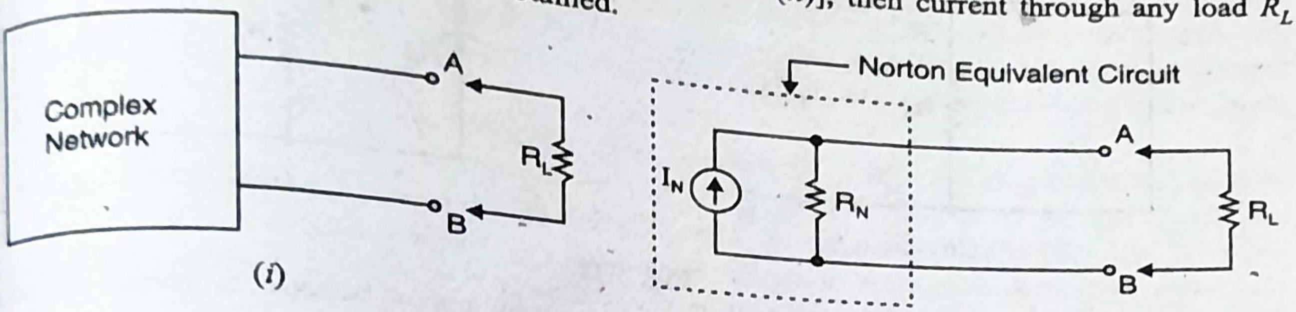
Norton's theorem is converse of Thevenin's theorem in the following respect. Norton equivalent circuit uses a current generator instead of voltage generator and the resistance R_N (which is the same as R_{Th}) in parallel with the generator instead of being in series with it.

When load R_L is connected between terminals A and B, then current in R_L is given by ;

$$I = \frac{E_{Th}}{R_{Th} + R_L}$$

11. Norton's Theorem

Fig. 3.10 (*i*) shows a network enclosed in a box with two terminals A and B brought out. The network in the box may contain any number of resistors and e.m.f. sources connected in any manner. But according to Norton, the entire circuit behind AB can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. 3.10 (*ii*). The resistance R_N is the same as Thevenin resistance R_{Th}. The value of I_N is determined as mentioned in Norton's theorem. Once Norton's equivalent circuit is determined [See Fig. 3.10 (*ii*)], then current through any load R_L connected across AB can be readily obtained.



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Any linear, bilateral network having two terminals A and B can be replaced by a current source of current output I_N in parallel with a resistance R_N.

(*i*) *The output I_N of the current source is equal to the current that would flow through AB when A and B are short-circuited.*

(*ii*) *The resistance R_N is the resistance of the network measured between A and B with load removed and the sources of e.m.f./current replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.*

Norton's theorem is converse of Thevenin's theorem in the following respect. Norton equivalent circuit uses a current generator instead of voltage generator and the resistance R_N (which is the same as R_{Th}) in parallel with the generator instead of being in series with it.

Illustration. Fig. 3.11 illustrates the application of Norton's theorem. As far as the circuit behind terminals AB is concerned [See Fig. 3.11 (*i*)], it can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. 3.11 (*iv*). The output I_N of the current generator is equal to the current that would flow when terminals A and B are short circuited as shown in Fig. 3.11 (*ii*). The load on the source when terminals AB are short-circuited is given by ;

$$R' = R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

$$\text{Source current, } I' = \frac{V}{R'} = \frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Short-circuit current, I_N = Current in R_2 in Fig. 3.11 (*ii*).

$$= I' \times \frac{R_3}{R_2 + R_3} = \frac{V R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

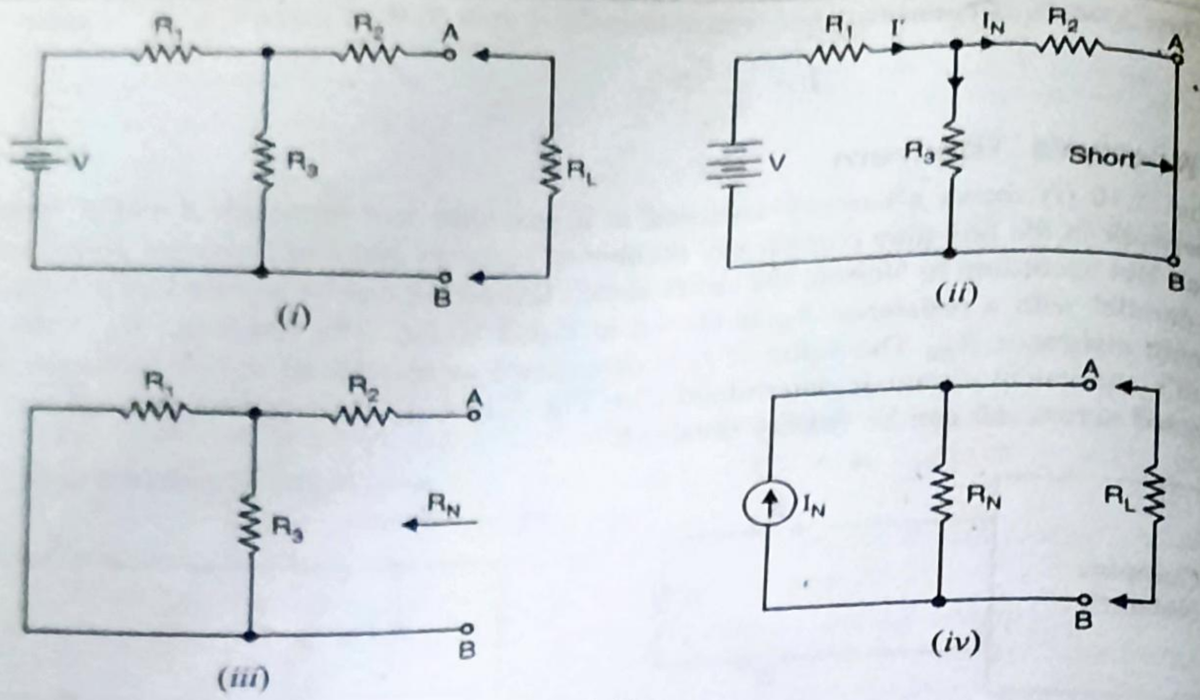

Fig. 3.11

To find R_N, remove the load R_L and replace battery by a short because its internal resistance is assumed to be zero [See Fig. 3.11 (*iii*)].

$$R_N = \text{Resistance at terminals } AB \text{ in Fig. 3.11 } (iii)$$

$$= R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Thus the values of I_N and R_N are known. The Norton equivalent circuit will be as shown in Fig 3.11 (*iv*).

12. Maximum Power Transfer Theorem

This theorem deals with transfer of maximum power from a source to load and may be stated as under :

In d.c. circuits, maximum power is transferred from a source to load when the load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all e.m.f. sources replaced by their internal resistances.

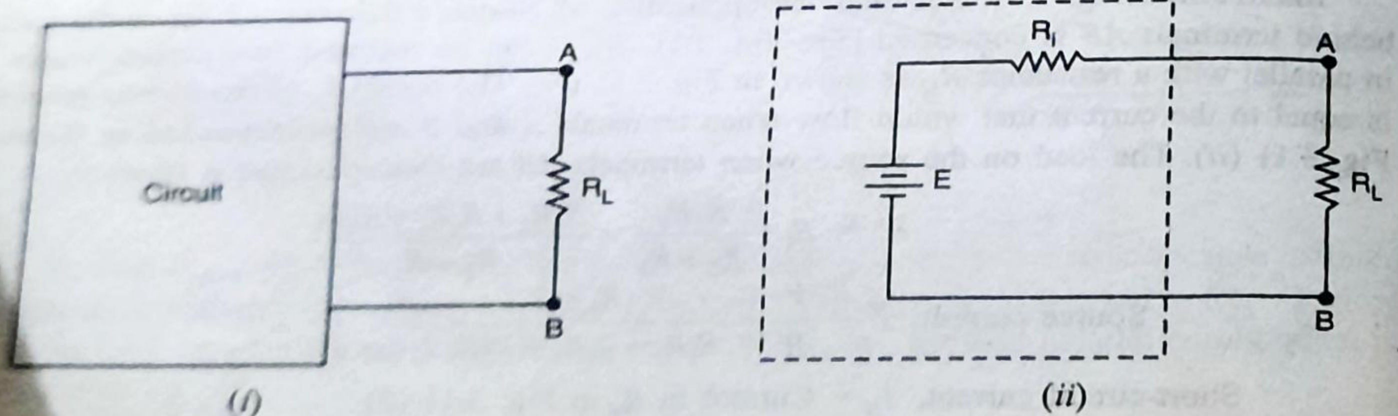

Fig. 3.12

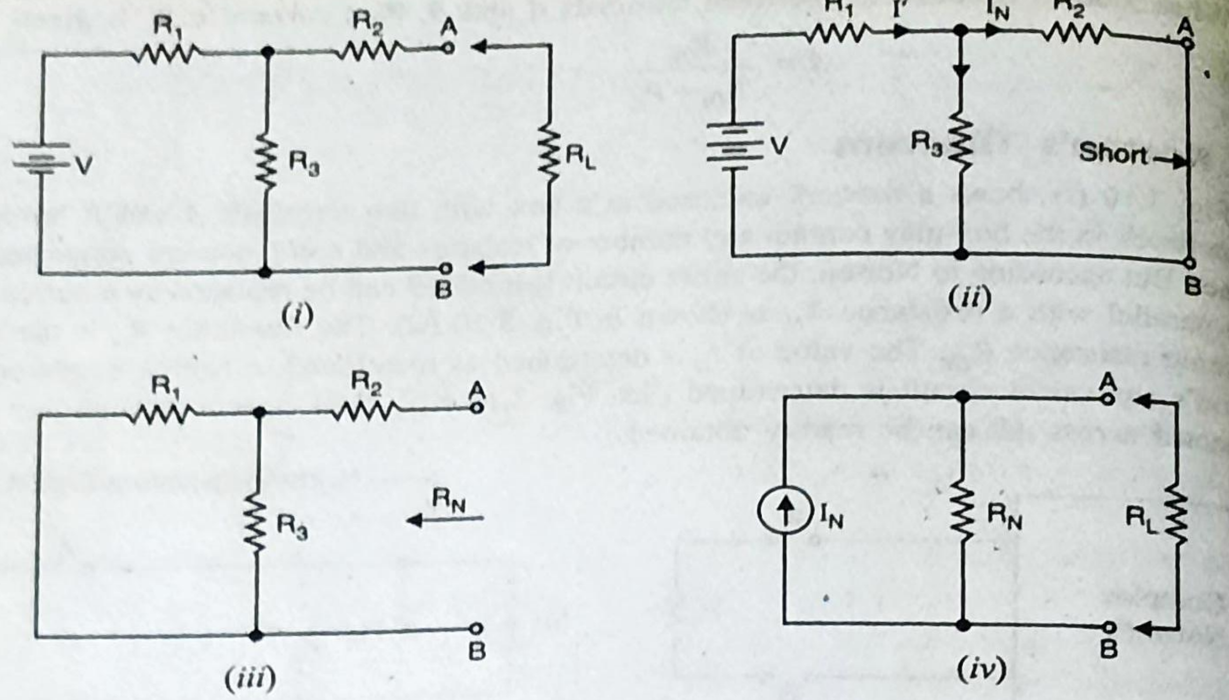

Fig. 3.11

To find R_N, remove the load R_L and replace battery by a short because its internal resistance is assumed to be zero [See Fig. 3.11 (*iii*)].

$$\therefore \quad R_N = \text{Resistance at terminals } AB \text{ in Fig. 3.11 } (iii)$$

$$= R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Thus the values of I_N and R_N are known. The Norton equivalent circuit will be as shown in Fig. 3.11 (*iv*).

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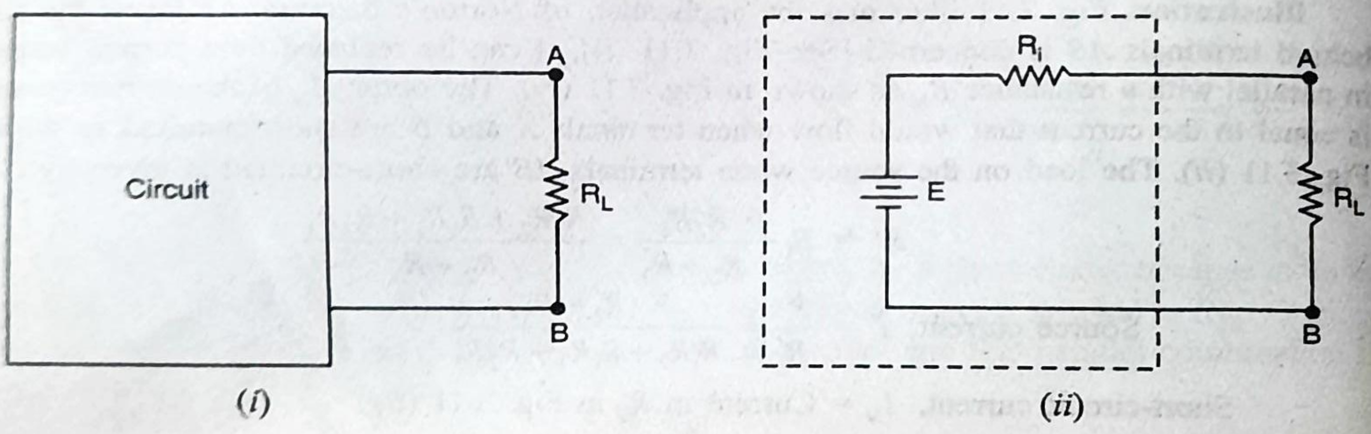

Fig. 3.12

Fig. 3.12 (*i*) shows a circuit supplying power to a load R_L. The circuit enclosed in the box can be replaced by Thevenin's equivalent circuit consisting of Thevenin voltage E (= E_{Th}) in series with Thevenin resistance R_i (=R_{Th}) as shown in Fig. 3.12 (*ii*). Clearly resistance R_i is the resistance measured between terminals *AB* with R_L removed and e.m.f. sources replaced by their internal resistances. According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when R_L is made equal to R_i, the Thevenin resistance at terminals *AB*. The proof of this theorem is left as an exercise for the reader.

Note. Under the conditions of maximum power transfer, the efficiency is only 50% as one-half of the total power generated is dissipated in the internal resistance R_i of the source.

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